Macro-Spatial Economics

Lecture 2: Facts and Classic Models (Allen-Arkolakis I)

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Why Quantitative Spatial Models?

From last week:

- Hsieh-Moretti: Spatial misallocation are costly in aggregate terms
- Housing constraints prevent efficient allocation
- Need framework to quantify spatial frictions

This week & next: Allen & Arkolakis (2025)

- **Today**: Motivation, facts, simple model, extensions
- **Next week**: Workhorse framework, taking to data, applications

Definition: "Quantitative"

Frameworks with rich interactions between locations that enable tight connection between theory and spatial data

Coverage Today

Introduction

- What are the big questions?
- Why does space matter?

Three motivating facts

Seminal models: Roback (1982), Krugman (1991)

Simple quantitative model

Extensions and microfoundations

Fact #1: Spatial Inequality is Enormous

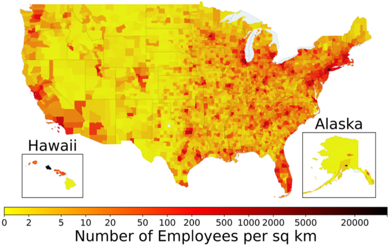
Population density across U.S. counties:

- Mountain West: < 1 worker per km^2
- Eastern seaboard: $> 1{,}000$ workers per km^2
- Several orders of magnitude difference

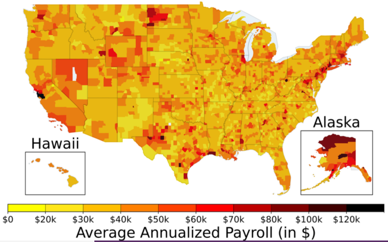
Output per worker:

- San Francisco, Manhattan, Seattle: $>$ \$100,000
- Counties nearby: $<$ \$50,000
- Total economic activity even more concentrated

(a) Workers per square kilometer



(b) Annual payroll per worker



Spatial Distribution is Persistent

Compare 1986 vs 2017:

- Distribution of workers across counties: remarkably stable
- Distribution of output across counties: remarkably stable
- Not a transitional phenomenon

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Implication

Need framework explaining persistent substantial spatial variation in where people live and how much they earn

Fact #2: Regional Trade is Important

Data: 2017 Commodity Flow Survey (70 MSAs)

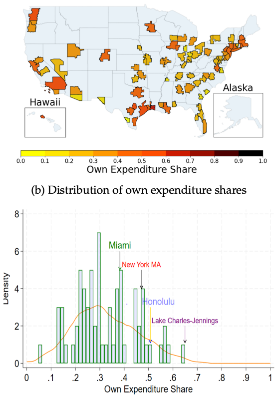
Own expenditure share: $\frac{\text{within-MSA spending}}{\text{total MSA spending}}$

- Nearly all MSAs spend majority on shipments from **other** MSAs
- Cross-MSA trade comprises majority of economic activity

Variation:

- Large MSAs (New York, Miami): own share $\approx$ 40%+
- Small Midwest MSAs: own share $\approx$ 25%

(a) Spatial variation in own expenditure shares



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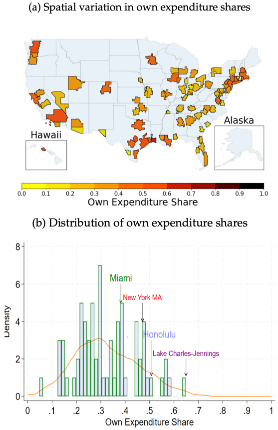


Implication

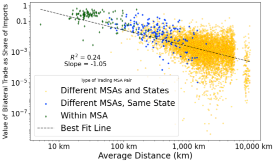
Need framework where different regions interact through flow of goods

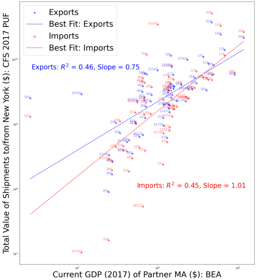
Fact #3: Gravity Holds

Trade declines systematically with distance:

- 250-500 km away: $\approx 10\%$ (10^{-1}) of <250 km
- >4000 km away: $\approx 1\%$ of <250 km

Size matters: Trade with NYC increases with partner GDP





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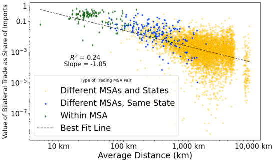

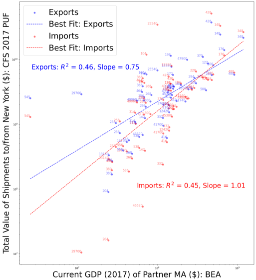

Implication

Framework must capture how distance and size determine degree of regional interaction through trade

Two Foundational Models

Rosen (1979) - Roback (1982):

- Location choice model
- Agents choose where to live
- No trade between locations

Krugman (1991) Core-Periphery:

- Trade between locations
- Monopolistic competition, increasing returns
- Agglomeration forces

Goal

Understand insights and limitations before building modern framework

Roback Model: Setup

N **locations**, measure $\bar{L}$ of identical, perfectly mobile agents

Welfare in location j:

$$W_j = w_j \times \bar{u}_j \times L_j^{\beta} \tag{1}$$

where:

- w_j: wage
- $\bar{u}_j$: innate amenity
- L_j: population
- $\beta < 0$: congestion externality

Free mobility: $W_j = W$ for all $j \in \mathcal{N}$

This gives **labor supply**:

$$L_j = w_j^{-1/\beta} \times \bar{u}_j^{-1/\beta} \times W^{1/\beta} \tag{2}$$

Upward sloping in wage (since $\beta < 0$); $\bar{u}_j$ outward shifter

Roback Model: Production

Production function:

$$Q_i = \frac{1}{1-\alpha} \bar{A}_i \bar{K}_i^{\alpha} L_i^{1-\alpha}$$

- $\bar{A}_i$: innate productivity
- $\bar{K}_i$: capital endowment
- α: capital share

Labor demand (FOC with respect to labor):

$$L_i = w_i^{-1/\alpha} \times \bar{A}_i^{1/\alpha} \bar{K}_i \tag{3}$$

Downward sloping (since $\alpha > 0$); $\bar{A}_i$ and $\bar{K}_i$ outward shifters

Roback Model: Equilibrium

Equating supply and demand yields:

$$L_i = \bar{u}_i^{\frac{1}{\alpha-\beta}} \times (\bar{A}_i \bar{K}_i^{\alpha})^{\frac{1}{\alpha-\beta}} \times W^{\frac{1}{\beta-\alpha}} \tag{4}$$

$$w_i = \bar{u}_i^{-\frac{\alpha}{\alpha-\beta}} \times (\bar{A}_i \bar{K}_i^{\alpha})^{-\frac{\beta}{\alpha-\beta}} \times W^{\frac{\alpha}{\alpha-\beta}} \tag{5}$$

$$W = \left[\sum_{i \in \mathcal{N}} \bar{u}_i^{\frac{1}{\alpha-\beta}} \times (\bar{A}_i \bar{K}_i^{\alpha})^{\frac{1}{\alpha-\beta}} / \bar{L}\right]^{\alpha-\beta} \tag{6}$$

with

$$\bar{L} = \sum_j L_j.$$

Insights:

- Population larger where amenities, productivity, capital better
- Wages higher where productivity/capital better, lower where amenities better
- Parameters α, β crucial for magnitude of effects

Roback Model: Limitations

Cannot explain Facts #2 and #3:

- Abstracts entirely from linkages through trade
- No concept of distance
- No heterogeneity in who trades with whom
- Economic activity in location i unaffected by other locations (except through scalar W)

Key Problem

Missing crucial element: the flow of goods between regions

Krugman Core-Periphery: Setup

N **locations**, each with measure N_i of firms producing differentiated varieties

Consumer problem: CES demand

$$\max_{j \in \mathcal{N}, \{c_{ij}(\omega)\}} \left[\sum_{i \in \mathcal{N}} \int_{\Omega_i} c_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \text{ s.t. } \sum_i \int_{\Omega_i} p_{ij}(\omega) c_{ij}(\omega) d\omega \leq w_j \tag{7}$$

where $\sigma > 1$ is the elasticity of substitution, Ω_i is set of firms in i
c_{ij} = consumption of good produced in i and sold in j

Results:

- Demand for variety ω in j: $y_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma} P_j^{\sigma-1} w_j$
- Indirect utility: $W_j = w_j / P_j$
- Price index: $P_j^{1-\sigma} \equiv \sum_i \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega$

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Inward Market Access

$P_j^{1-\sigma} \equiv \sum_i \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega$ measures how cheaply location j can source goods — lower prices from nearby producers.

Krugman Model: Production and Pricing

Firms:

- Use labor only
- Monopolistic competition
- Iceberg trade costs: $\tau_{ij} \geq 1$

Optimal pricing:

$$p_{ij} = \frac{\sigma}{\sigma - 1} \frac{w_i}{A_i} \tau_{ij} \tag{10}$$

Total firm demand: $y_i(\omega) = \sum_j \frac{\left(\frac{\sigma}{\sigma-1}\frac{w_i}{A_i}\tau_{ij}\right)^{1-\sigma}}{P_j^{1-\sigma}} E_j$ with $E_j = w_j L_j$

Krugman Model: Production and Pricing

Firm profits: $y_i(\omega)/\sigma$

Entry: Fixed cost f_i^e workers. Zero profit condition:

$$w_i f_i^e = \frac{1}{\sigma} \sum_j \left(\frac{\sigma}{\sigma - 1} \frac{w_i}{A_i} \tau_{ij} \right)^{1-\sigma} E_j / P_j^{1-\sigma}$$

Market clearing:Total labor income equal to the sum of sales:

$$w_i = \frac{N_i}{L_i} \frac{1}{\sigma} \sum_j \left(\frac{\sigma}{\sigma - 1} \frac{w_i}{A_i} \tau_{ij} \right)^{1-\sigma} E_j / P_j^{1-\sigma}$$

Equilibrium number of firms: $N_i = L_i / (\sigma f_i^e)$

Krugman Model: Labor Supply and Demand

Labor supply (from welfare equalization):

$$w_i = W \left[\sum_j \frac{L_j}{\sigma f_j^e} \left(\frac{\sigma}{\sigma - 1} \frac{w_j}{A_j} \tau_{ji} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \tag{14}$$

Labor demand (from zero profit):

$$w_i = A_i^{\frac{\sigma-1}{\sigma}} \left(\frac{\sigma}{\sigma - 1} \right)^{\frac{1-\sigma}{\sigma}} \left(\frac{1}{\sigma f_i^e} \right)^{1/\sigma} \times \left[\sum_j \tau_{ij}^{1-\sigma} P_j^{1-\sigma} E_j \right]^{1/\sigma} \tag{15}$$

Key observation: Both perfectly inelastic to local labor L_i!

- Supply: no congestion force directly
- Demand: competition offset by agglomeration (firm entry)
- Instead, depend on **market access**

Market Access in Krugman Model

Inward market access $P_j^{1-\sigma}$:

- Locations with better inward MA face lower prices
- Closer to low-cost producers
- Attracts workers (higher purchasing power)

Outward market access $\Pi_i^{1-\sigma} \equiv \sum_j \tau_{ij}^{1-\sigma} P_j^{1-\sigma} E_j$:

- Locations with better outward MA face higher demand
- Closer to high-demand destinations
- Attracts firms (higher profits)

Problem

Perfectly inelastic supply and demand $\Rightarrow$ complete concentration in one location (without ad-hoc fix)

Krugman Model: Key Insights

Innovation 1: Market access from trade linkages

- Crucial for spatial economy (Facts #2, #3)
- Still cornerstone of modern models

Innovation 2: Endogenous agglomeration (firm entry)

- Economic agglomerations exacerbate innate differences
- Trade costs matter: lower costs $\Rightarrow$ affects concentration

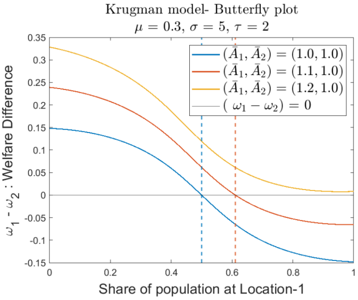
Innovation 3: Multiple equilibria possible

- Famous "Tomahawk graph"
- Below threshold trade costs: 3 equilibria (symmetric + 2 concentrated)
- Raises question: is multiplicity real or artifact?

Let's examine these insights with simulations...

Insight 1: Innate Differences and Agglomeration

Figure 6: Two locations, vary A_1 with $A_2 = 1$ fixed



Results:

- $A_1 = A_2 = 1$: symmetric, equal population
- $A_1 = 1.1$ (10% higher): population share in location 1 $>$ 20% higher
- Disproportionate response to small productivity differences
- $A_1 = 1.2$: welfare always positive $\Rightarrow$ corner solution (all in location 1)

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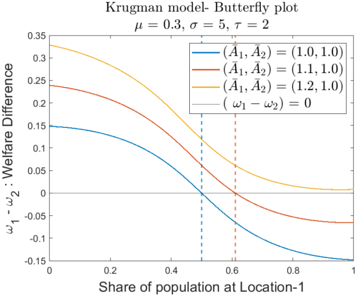


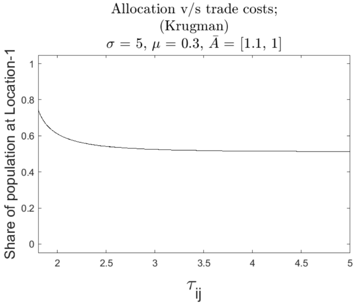
Insight

Endogenous agglomeration force **exacerbates** innate differences

Insight 2: Market Access and Economic Outcomes

Key findings:

- Very high τ: locations independent, location 1 slightly larger
- As τ decreases: population difference grows dramatically
- At $\tau < 2$: location 1 share > 0.75



Mechanism:

- Lower trade costs $\Rightarrow$ better producer market access
- Higher labor demand, higher wages
- Attracts population through entry
- Creates even stronger agglomeration
- Two forces coalesce at low trade costs

Krugman Model: Limitations

Problems:

- Equilibrium may not exist (complete concentration)
- May be unrealistic even with small differences in fundamentals
- Strong, inflexible agglomeration force (only through entry)
- Large responses to small trade cost changes
- Exacerbated with many locations, complex geography

Next Step

Combine richness of trade linkages (Krugman) with tractability of location choice (Roback)

Building the Modern Framework

Goal: Combine best of both models

- Trade linkages and market access (from Krugman)
- Well-behaved supply and demand (from Roback)
- Flexible agglomeration forces (not just entry)

Based on: Allen & Arkolakis (2014)

Key innovation

Model agglomeration through productivity and amenity externalities (parameters α, β) rather than just firm entry

Setup: Geography

Locations: Compact set $\mathcal{N}$, indexed by $i, j \in \mathcal{N}$

Each location i endowed with:

- Innate productivity: $\bar{A}_i$
- Innate amenity: $\bar{u}_i$
- Trade technology: $\tau_{ij} \geq 1$ (iceberg costs)
  - $\tau_{ii} = 1$
  - $\tau_{ij} > 1$ for $i \neq j$

Population: Measure $\bar{L}$ of perfectly mobile agents

- Choose where to live
- Produce and consume there

Note: Particular value of $\bar{L}$ not important (won't affect shares)

Geography

The set of $\{\bar{A}_i\}_{i \in \mathcal{N}}$, $\{\bar{u}_i\}_{i \in \mathcal{N}}$, $\{\tau_{i,j}\}_{i,j \in \mathcal{N}}$

Production

Armington assumption: Each location produces unique variety

- Labor only factor of production
- Perfect competition
- Price equals marginal cost:

$$p_{ij} = \tau_{ij} \frac{w_i}{A_i} \tag{16}$$

where w_i is wage, A_i is total productivity

Important: $A_i \neq \bar{A}_i$ (total vs. innate)

- Relationship defined below through agglomeration

Consumption

Agent $\ell \in [0, \bar{L}]$ solves:

$$\max_{j \in \mathcal{N}, \{c_{ij}\} \geq 0} \left[\sum_{i \in \mathcal{N}} c_{ij}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \times u_j \quad \text{s.t.} \quad \sum_i p_{ij} c_{ij} \leq w_j$$

where:

- $\sigma \geq 0$: elasticity of substitution
- u_j: total amenity
- p_{ij}: price of good from i in j

Again: $u_j \neq \bar{u}_j$ (total vs. innate)

Demand and Indirect Utility

Optimization yields:

Indirect utility in location i:

$$W_i = \frac{w_i u_i}{P_i} \tag{18}$$

where $P_i = \left[\sum_{j \in \mathcal{N}} p_{ji}^{1-\sigma}\right]^{1/(1-\sigma)}$ is the inward market access (price index)

Quantity consumed by an agent residing in j:

$$c_{ij} = p_{ij}^{-\sigma} P_j^{\sigma-1} w_j$$

Bilateral trade flows:

$$X_{ij} = p_{ij}^{1-\sigma} P_j^{\sigma-1} w_j L_j$$

Gravity Equation

Substitute prices into trade flows:

$$X_{ij} = \tau_{ij}^{1-\sigma} \left(\frac{w_i}{A_i} \right)^{1-\sigma} P_j^{\sigma-1} w_j L_j \tag{21}$$

Interpretation when $\sigma > 1$, i.e. goods are substitute:

- Trade **decreases** in τ_{ij} (trade costs)
- Trade **decreases** in w_i / A_i (cost of production in origin)
- Trade **increases** in $w_j L_j$ (total income in destination)
- Trade **increases** in P_j (harder to get goods from elsewhere)

Success

This matches Fact #3 (gravity) from the data!

Agglomeration Forces: The Key Innovation

Total productivity as function of population:

$$A_i = \bar{A}_i L_i^{\alpha} \tag{22}$$

- $\alpha > 0$: agglomeration (entry, spillovers, thick markets)
- $\alpha < 0$: congestion (fixed factors like land/capital)
- $\alpha = 0$: no externalities

Total amenity as function of population:

$$u_i = \bar{u}_i L_i^{\beta} \tag{23}$$

- $\beta > 0$: agglomeration (variety, better services)
- $\beta < 0$: congestion (housing costs, crowding)
- $\beta = 0$: no externalities

These are "reduced form" capturing many possible mechanisms

Why Agglomeration/Congestion Matter

Fixes Krugman's problem:

- Labor supply and demand now depend on local labor
- Not just on market access
- Creates well-behaved equilibrium

Creates realistic spatial patterns:

- Can match observed distribution
- Can be calibrated to data
- Flexible enough for policy analysis

Parameters α, β crucial:

- Determine how geography affects equilibrium
- Affect both quantitative and qualitative predictions
- Must be carefully estimated

Summary: What We Learned Today

Three key empirical facts:

1. Enormous spatial inequality in population and output
2. Regional trade is substantial and follows gravity
3. Distance and size determine interaction intensity

Two seminal models:

- Roback: Location choice, no trade
- Krugman: Trade and agglomeration, but knife-edge equilibria

Simple quantitative model:

- Combines trade linkages with well-behaved labor markets
- Agglomeration in production, congestion in amenities
- Tractable equilibrium with market access

Next Week: Extensions and Applications

Lecture 3: Extensions to the simple model

- Equilibrium
- Special cases: no externalities, two locations
- Alternative microfoundations (Ricardian, housing, etc.)
- Multiple sectors and input-output linkages

Reading:

- Allen & Arkolakis (2025) Sections 4-5
- Focus on extensions and robustness