

# Macro-Spatial Economics

## Lecture 10: Sorting

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# Where We Are: A Course in Three Questions

- **Why does space matter?** HM19: housing constraints cost 2–9% of GDP
- **How do we measure it?** AA Lectures 2–4: gravity, hat algebra, welfare
- **Who bears the cost?** DR/HM24: landlords capture gains; incidence via ownership

**Then adjustment and amplification (L6–9):** migration, commuting, shift-share IVs, housing over the cycle, transition dynamics

## The missing question

Every lecture took the initial sorting of workers as given. No paper asked: *what produces the equilibrium sorting, and is it efficient?* That is the question Lecture 10 opens.

# The Puzzle: Productive Cities Are Ageing Fast

## England, 2002–2019

- TTWAs in the **top wage decile**: mean working age rose by **1.2 years** (national avg: 0.7)
- Young share (25–34) in high-wage TTWAs **fell by 2.0 p.p.** (national: 0.3 p.p.)
- These same locations are the **most expensive** housing markets

## The puzzle

If productive cities pay higher wages *and* offer steeper career growth, why are young workers being priced out? ⇒ **Age-varying housing costs.**

# Fact 1: Age-Specific Housing Costs — LTV Distribution

**Data:** English Housing Survey,  $N = 67,268$  households.

**LTV distribution by age group (Table 1, GP 2026):**

Age group	Cash/ $\leq 60\%$	60–70%	70–80%	80–90%	$\geq 90\%$
25–34	0.117	0.077	0.178	0.364	<b>0.264</b>
35–44	0.233	0.113	0.163	0.270	0.221
45–64	0.443	0.110	0.127	0.187	0.133
65+	0.685	0.046	0.068	0.123	0.079

- Nearly **two-thirds** of young homebuyers had  $LTV > 80\%$ ; only 11% in the safe category
- Higher LTV  $\Rightarrow$  higher mortgage rate premium  $\Rightarrow$  higher effective housing cost

# Fact 1: Regression Evidence (Table 2, GP 2026)

$$\log H_{i,j,t} = \gamma^g \text{Age}_{i,j,t}^g + \beta_1 \text{Term} + \beta_2 \log \text{Inc} + \beta_4 \text{LTV} + \alpha_j + \theta_t + \varepsilon_{i,j,t}$$

## Key results (reference group: 25–34)

- Age 35–44:  $-0.083^{***}$ ; Age 45–64:  $-0.136^{***}$  (8–14% lower costs)
- Adding LTV controls: gap falls by **35%**  $\Rightarrow$  LTV is the mechanism
- London/South-East interaction: additional  $-0.053$  to  $-0.066$  (gap 7 p.p. larger in productive regions)

Young workers pay **13–19%** more in housing costs, everything else equal.

## Fact 2: Age-Specific Location Returns

**Data:** ASHE, 2004–2019,  $N = 2.25\text{M}$ . “Stayers” = same TTWA for  $\geq 3$  years.

### Two findings (Figures 1–4, GP 2026)

- ① **Wage levels:** spatially correlated across age groups (high in London/SE for all)
- ② **Wage growth:** young (25–34) concentrated in productive South-East; old (50–64) low and spatially homogeneous everywhere

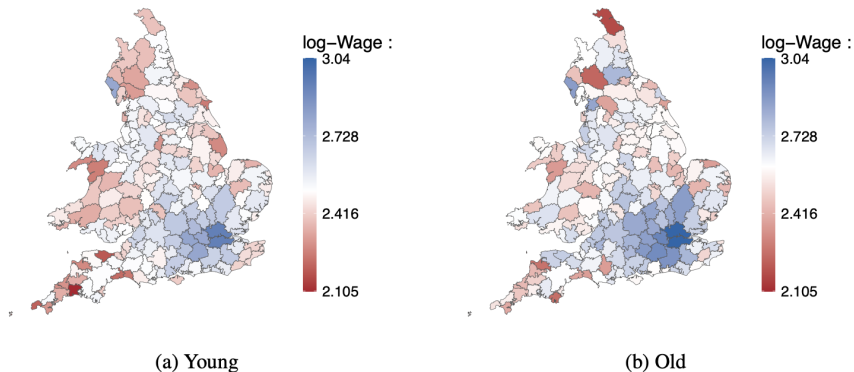
Cross-TTWA correlation of young and old wage *growth*  $\approx 0$

### Implication

Exclusion from productive cities costs the young *more* — they lose the steepest phase of human capital accumulation.

## Fact 2a: Wage Maps — Young vs. Old

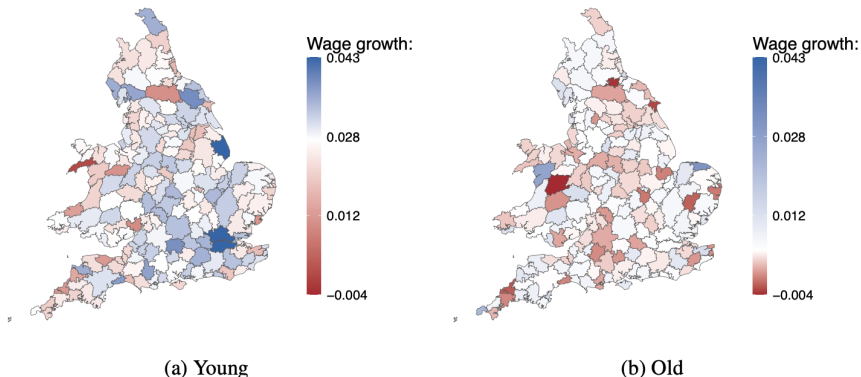
Figure 1: Average Wage for stayers by age



*Left: wage level, young (25–34). Right: old (50–64). Wage levels are concentrated in the South-East for both age groups.*

## Fact 2b: Wage Growth Maps — Young vs. Old

Figure 2: Wage growth for stayers by age

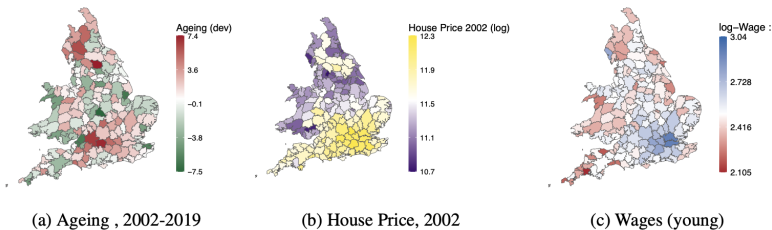


*Left: annualised wage growth, young (25–34). Right: old (50–64). Young wage growth is concentrated in the South-East; old wage growth is uniformly low.*

## Fact 3: Differential Ageing of Productive Areas (Table 3)

$$\text{Aging}_{j,t,h} = \Delta h \left( \frac{N_{j,t}^{50-64} - N_{j,t}^{25-34}}{N_{j,t}^{16-64}} \right)$$

Figure 5: Ageing, House Price, and Value Added in England and Wales



A 10% wage advantage  $\Rightarrow \approx 0.6$  p.p. faster ageing. Young workers are displaced from high-wage locations; older workers with lower housing costs concentrate there.

# The Mechanism: Why the Market Gets It Wrong

The three facts point to a single story:

- ① Young workers have **higher housing costs** than old workers in the same location, because they borrow at higher LTV and pay a mortgage rate premium (Fact 1)
- ② Productive cities offer **steep wage growth for the young** but not the old — the returns to being in a productive city are age-specific (Fact 2)
- ③ As a result, old workers — who face lower housing costs *and* care less about future wage growth — **outbid the young** for space in productive cities. These cities age disproportionately (Fact 3)

## Why is this inefficient?

The market does not price the **dynamic externality**: a young worker excluded from a productive city loses the steepest phase of human capital accumulation — a loss that persists for the rest of their career and across generations. An old worker displacing them gains current utility but generates no future skill spillovers. The market equilibrium has too many old and too few young in productive locations.

# The Two-Location OLG Model: Environment

**Two-period OLG.** Young ( $Y$ ) and old ( $O$ ); two locations: city ( $C$ ), rural ( $R$ ).

- Productivity:  $Z_C > Z_R$ . Wages =  $Z_j$  (competitive).
- Age-specific amenities  $\mu_j^g$ ; returns to experience  $e_j$  ( $e_R = 0$ ,  $e_C > 0$ )
- Old skill:  $s^O(j^Y) = s^Y(1 + e + e_j)$  — depends on where young

## The key friction: age-specific housing costs

Living in  $j$  costs  $\psi^g P_j$ .  $\psi^Y = 1$ ,  $\psi^O = (1 - \psi)$  with  $\psi \in (0, 1)$ .

Young pay full price; old pay  $(1 - \psi)$  less.  $\psi$  measures the intergenerational friction.

# Workers' Problem and Equilibrium

**Old** (static):  $V^O = \max_j \{ \mu_j^O + w_j s^O(j^Y) + \tau_t - (1 - \psi)P_j + \varepsilon_{j,t} \}$

**Young** (dynamic):

$V^Y = \max_j \{ \mu_j^Y + w_j s^Y + \tau_t - P_j + \varepsilon_{j,t} + \mathbb{E}[V^O(s^O(j), \varepsilon_{t+1})] \}$

## Key asymmetry

Young pay  $P_j$ ; old pay  $(1 - \psi)P_j$ . Old workers have higher income from accumulated skills *and* lower housing costs — crowding out the young.

Idiosyncratic shocks  $\varepsilon_{i,j} - \varepsilon_{i,j'} \sim U(-\sigma, \sigma)$ . Unique interior steady state under Assumptions S.1–S.5.

# Proposition 1: Market Equilibrium (Equation 1, GP 2026)

**Proposition 1.** Steady-state share of young in the city:

$$\tilde{\phi}_C^Y = \frac{1}{2} + \frac{1}{D} \left[ \underbrace{(1 - \psi)\Delta\mu^Y - \Delta\mu^O}_{\text{Net amenity pull}} + \underbrace{(1 - \psi)Z_R e_C - e\Delta Z}_{\text{HC incentives (scaled by friction)}} - \underbrace{\psi \Delta Z \left(1 + \frac{e_C}{2}\right)}_{\text{Housing cost distortion}} \right]$$

- **Housing distortion:**  $-\psi\Delta Z(1 + e_C/2)$  — the **intergenerational congestion wedge**; vanishes only when  $\psi = 0$
- Misallocation grows with city productivity  $\Delta Z$  and city learning premium  $e_C$

## Proposition 2: Planner's Allocation and Efficiency

**Proposition 2.** The planner's share of young in the city:

$$\hat{\phi}^Y = \tilde{\phi}^Y + \psi \left[ \underbrace{\frac{\Delta\mu^Y + \Delta Z - 2\sigma(\tilde{\phi}^Y - \frac{1}{2})/\mathcal{D}_0}{\mathcal{D}_0}}_{\text{Static sorting wedge}} + e_c \underbrace{\frac{Z_R + \Delta Z(\frac{1}{2} - (\tilde{\phi}^Y - \frac{1}{2})/\mathcal{D}_0)}{\mathcal{D}_0}}_{\text{Dynamic HC wedge}} \right]$$

### Key results

- $\psi = 0 \Rightarrow \hat{\phi}^Y = \tilde{\phi}^Y$ : market is **efficient** iff  $\psi = 0$
- $\psi > 0 \Rightarrow \hat{\phi}^Y > \tilde{\phi}^Y$ : planner puts *more* young in the city
- **Static wedge**: old with lower city preference displace young due to cost advantage
- **Dynamic wedge**: young priced out miss critical HC accumulation — persistent loss

# Quantitative Model: From Two Locations to 52 TTWAs

- ① **Geography:** 52 UK TTWAs (merged from 173; on average 76% internal commuting cohesion)
- ② **Life cycle:** multiple working periods and retirement
- ③ **Heterogeneity:** ex-ante skill  $s$  and wealth  $b$ ; endogenous homeownership
- ④ **Endogenous housing costs:** LTV-dependent mortgage rates microfound  $\psi$

## State vector and skill accumulation

$\Omega := (b, h, j, s, a)$  — liquid wealth, housing, location, skill, age

Inherits **shock-age timing** from DUE (Lecture 9); adds OLG structure, LTV mechanism, and intergenerational HC externality

# Location-Specific Skill Accumulation (Equation 3, GP 2026)

Following Lhuillier (2024):

$$e_j(s_i^a, a) = \underbrace{\gamma_1(a) \ln Z_j}_{\text{Direct TFP}} + \underbrace{\gamma_2(a) \ln \bar{s}_j}_{\text{Peer skill}} + \underbrace{\gamma_3(a) \ln Z_j \cdot \ln s_i^a}_{\text{Skill-TFP compl.}} + \underbrace{\gamma_4(a) \ln \bar{s}_j \cdot \ln s_i^a}_{\text{Peer-skill compl.}} \quad (3)$$

- Parameters  $\gamma_k(a)$  are **age-varying**: learning capacity differs across the life cycle
- Complementarities ( $\gamma_3, \gamma_4$ ): high-skill workers gain *more* from productive locations
- Exclusion from productive cities is *especially costly for the young*: steepest learning curve

# Housing Market and Endogenous $\psi$

## Tenure choice (at shock ages):

- **Renting:** flexible, no wealth required
- **Owning:** utility premium  $\chi \geq 1$ ; costs include maintenance  $\delta p_j h$  and transaction cost  $\Psi p_j h$  when selling

## Financial frictions:

- LTV constraint:  $b \geq -M^P p_j h$
- Income constraint:  $b \geq -M^Y y(j, s, a)$
- Mortgage rate:  $q(b, j, h) = q + \mathbf{1}_{[b < 0]} \cdot f\left(\frac{-b}{p_j h}\right)$

## How $\psi$ becomes endogenous:

In the toy model, young pay  $P_j$  and old pay  $(1 - \psi)P_j$  with  $\psi$  exogenous. Here, the mechanism is microfounded:

- 1 Young have low wealth  $b$
- 2  $\Rightarrow$  LTV constraint binds at higher  $b/p_j h$
- 3  $\Rightarrow$  higher mortgage rate premium  $f(\cdot)$
- 4  $\Rightarrow$  higher effective housing cost

The age-cost ratio  $\psi(a)$  emerges endogenously from the wealth distribution — exactly what Fact 1 documented.

# Policy: The Council Tax Reform

## Design

Reduce housing costs for **young workers (21–34)** in the **7 most productive TTWAs**; financed by a modest increase for **workers aged 40+**. Revenue-neutral.

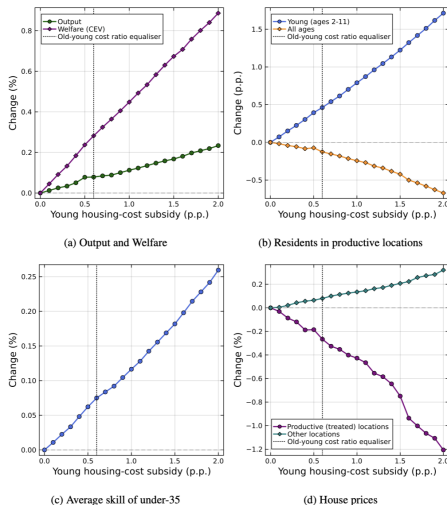
- Young discount: **0.6% of house price** (= 4.1% of rent)
- Old surcharge: 0.12% of property value

## Design principle

Not a place-based policy (expanding city size). An **age-place-based** intervention correcting the intergenerational *composition* within productive cities.

# Policy Results: Age-Based Policy

Figure 8: Policy effects of a place-age-based tax policy, by policy size



## Headline numbers

- Output: **+0.1%**; Welfare: **+0.3%** (CE)
- Young share rises: 29.8% → 30.3%
- Population *falls*: 21.9% → 21.8%

**Composition, not size:** gains from changing *who* is in productive cities, not making them bigger. Financing via 40–64 preserves 75–90% of gains.

# Small Policies, Large Effects

## Surprisingly large output gains

- Output: **+0.1%**; a larger subsidy (2% of house value) doubles the gains
- A 1% uniform TFP increase yields  $\sim 1\%$  output — this policy delivers  **$\approx 10\%$  of that** at zero fiscal cost
- Driven by **human capital accumulation**, not factor reallocation

## Not agglomeration

- Residents in productive TTWAs **fall** ( $-0.1$  p.p.)
- Output rises *despite* depopulation
- Young sort **better over the life cycle**: more spend learning years in the city, then move out
- Corrects *who* lives where *when*, not how many

## Policy debate

Standard urban policy targets expanding productive cities (Hsieh & Moretti). GP shows a complementary channel: correcting **age-specific frictions** improves human capital without requiring more people in expensive areas.

## FG Step 1: The Prior View on Place-Based Policies

Same AA structure from Lectures 2–4, plus one critical assumption:

- Agglomeration in productivity:  $z_j = Z_j L_j^{\gamma_P}$ , congestion in amenities:  $a_j = A_j L_j^{\gamma_A}$
- Utility:  $u_j = a_j \cdot c_j$ ; free mobility equalises  $u_j$  across locations
- **No spatial transfers:**  $c_j = z_j$  — no federal taxes, no Social Security, no intergovernmental transfers

Under  $c_j = z_j$ :  $u_j = A_j Z_j L_j^{\gamma_P + \gamma_A}$ . Free mobility + constant elasticities  $\Rightarrow$  average returns equalised  $\Rightarrow$  marginal returns equalised.

### Result (Glaeser & Gottlieb 2008; Kline & Moretti 2014)

Moving a worker from  $j$  to  $i$  gains agglomeration in  $i$  but loses it in  $j$  — a wash.  
**The market is efficient. No role for place-based policy.**

## FG Step 2: What Happens When $c_j \neq z_j$

$c_j = z_j$  is **counterfactual**. Governments redistribute across space (FG Figure I): net transfers negative in high-wage cities, positive in low-wage cities.

Without transfers: compensating differentials  $\Rightarrow$  high-amenity cities have low  $z_j$ . Workers equalise utility but **do not maximise output**. With transfers: the planner can move workers to high- $z$  locations and compensate for the amenity loss.

$$\frac{du}{u} = (\gamma^P + \gamma^A)(z_i - z_j) \frac{dL}{Y}$$

Gains whenever  $z_i \neq z_j$ , which holds whenever  $a_i \neq a_j$ .

Calibrated to 209 US MSAs: welfare gain = **0.06%**. Why so small? With one worker type, the only margin is city *size* — and free mobility already gets sizes roughly right.

## FG Step 3: From 0.06% to 4%

With one type: only city *sizes* to adjust. With two types: the skill **mix** within each city. The market cannot internalise cross-type spillovers  $\Rightarrow$  the mix can be badly wrong.

Spillover elasticities  $\gamma_{\theta, \theta'}^P, \gamma_{\theta, \theta'}^A$  are **type-pair specific**. Calibrated from Diamond (2016) and Ciccone & Hall (1996):

**Productivity**  $\gamma_{\theta, \theta'}^P$ :

- Skilled generate larger spillovers on *everyone* (incl. unskilled)

**Amenity**  $\gamma_{\theta, \theta'}^A$ :

- Skilled  $\rightarrow$  positive; unskilled  $\rightarrow$  negative (congestion)

Too much skill segregation: skilled abundant in big cities where marginal cross-spillover is low. Moving some to small cities  $\Rightarrow$  high-value spillovers on unskilled. If  $\gamma_{\theta, \theta'}^P = \gamma^P$  for all pairs: back to 0.06%. The **4% gain comes entirely from heterogeneous spillovers**.

## FG Step 4: The Efficiency Condition

**Proposition 1** — For all locations  $j$  and types  $\theta$ :

$$\underbrace{W_j \frac{dN_j}{dL_j^\theta}}_{\text{marginal product of type } \theta \text{ in } j} + \underbrace{\sum_{\theta'} \frac{L_j^{\theta'}}{L_j^\theta} \gamma_{\theta, \theta'}^A x_j^{\theta'}}_{\text{amenity spillovers on others in } j} = \underbrace{x_j^\theta}_{\text{what } \theta \text{ consumes}} + \underbrace{E^\theta}_{\text{opp. cost of } \theta \text{ elsewhere}}$$

**Left side** (marginal social benefit):

- $W_j$  = wage per efficiency unit in  $j$
- $N_j$  = aggregate efficiency labour in  $j$
- $\gamma_{\theta, \theta'}^A$  = amenity spillover of  $\theta$  on  $\theta'$

**Right side** (marginal social cost):

- $x_j^\theta = w_j^\theta + b^\theta \Pi + t_j^\theta$   
wage + portfolio share  $\times$  profits + transfer
- $E^\theta$  = shadow value of type  $\theta$  (same everywhere)

**Why expenditure, not wages?**

Transfers create a wedge: two workers, same wage, different  $t_j^\theta \Rightarrow$  different resource cost. Spatial analogue of Hsieh & Klenow (2009).

## FG Step 5: Optimal Transfers

The planner solves a **Pareto problem**: maximise  $u^\theta$  s.t.  $u^{\theta'} \geq \underline{u}^{\theta'}$ . The market is inside the frontier  $\Rightarrow$  Pareto improvements exist.

### Proposition 2 — Under constant elasticities

Optimal transfers = **spatial component**  $s_j^\theta$  + **distributional component**  $T^\theta$ :

- $s_j^\theta$ : labour income subsidy, varies by type *and* location — subsidise  $\theta$  where it generates large spillovers and is scarce
- $T^\theta$ : lump-sum, varies only by type — handles equity

### Principle of targeting

Spatial subsidies fix the spillover externality. Lump-sum transfers fix distribution. Cleanly separated — no blunt instruments needed.

## FG Step 6: What the Optimum Looks Like — and Why

**Why more mixing?** Skilled workers in large cities generate own-type spillovers, but the marginal value is low — they are already abundant there. Moving some to small cities generates **high-value cross-spillovers** on unskilled. The optimum trades own-type agglomeration for cross-type spillovers.

**Applied to 209 US MSAs (2007):**

- Large cities should be **smaller**; **11%** reallocated
- Urban skill premium **vanishes**
- City size: poor predictor ( $R^2 = 15\%$ )

Specification	Gain (%)
Benchmark	<b>4.0</b>
Homogeneous	<b>0.06</b>
High cross-amenity	5.6
Low amenity	2.8

### The 65× gap

0.06% → 4.0%: the entire gain comes from correcting *who* lives where, not how many. Inefficient **sorting**, not aggregate misallocation.

## FG: Actual vs. Optimal Transfers

The actual US transfer system has the **right sign** (redistribution from high-wage to low-wage cities) but the **wrong scale**:

	Optimal slope	Actual slope	
Low-skill	-0.095	-0.020	( $\approx 5\times$ too flat)
High-skill	-0.050	-0.002	( $\approx 25\times$ too flat)

### Why would high-skill workers in NYC accept higher taxes?

Because the policy is a **tax-and-transfer package**. The spatial component  $s_j^\theta$  taxes them more in high-wage cities. But the lump-sum  $T^\theta$  and higher aggregate profits  $\Pi$  (from better sorting) compensate. At the Pareto-frontier point where both types gain equally, the common welfare gain is **4%**. The GE gains from improved sorting expand the pie enough to make everyone better off.

## FG Step 7: From FG Back to GP

### GP as a special case of FG's Proposition 2

FG: whenever types generate different spillovers, first-best requires **type-specific spatial transfers**. GP applies this principle when:

- Type  $\theta = \text{age}$ ; spillover = **human capital accumulation**  $e_j(s_i^a, a)$
- Instrument = **age-varying housing taxes** in high-spillover TTWAs

### What GP adds beyond FG

FG is **static**: inefficiency arises from misallocated spillovers *today*. GP is **dynamic**: the young worker excluded from a productive city loses human capital that would have accumulated over the rest of their career. This **dynamic HC wedge** — a persistent, intergenerational loss — is invisible in FG's static framework. GP shows that the cost of misallocation is larger than a static model suggests.

# The Complete Course Arc

L	Paper	Contribution
1	HM (2019)	Housing constraints $\Rightarrow$ 2–9% GDP loss
2–4	AA (2025)	Measurement: gravity, hat algebra, welfare accounting
5	DR + HM24	Incidence: landlords and homeowners capture the gains
6	MRR + Noto	Adjustment: migration and commuting as correction margins
7	Spatial econometrics	Shift-share IV, Conley SEs, empirical toolkit
8	M&S + BHO	Amplification: housing transmits aggregate shocks
9	DUE + CDP	Transition: adjustment is slow, costly, distributional
<b>10a</b>	<b>GP (2026)</b>	<b>Sorting is inefficient: intergenerational congestion</b>
<b>10b</b>	<b>FG (2020)</b>	<b>Optimal policy: type-specific transfers; 4% welfare gain</b>

## The course's central answer

The spatial distribution is inefficient. The sources are multiple, costs large, and optimal policy must be targeted to the friction and the margin.

# Three Sources of Misallocation, Three Instruments

Source	Mechanism	Instrument
Housing supply constraints	Cities cannot expand; rents too high	Relax zoning (HM19)
Heterogeneous sorting externalities	Workers don't internalise spillovers	Type-specific transfers $t_j^\theta$ (FG)
Intergenerational congestion	Young priced out of high-learning cities	Age-place-based reform (GP)

## Do these interact?

Each instrument is necessary; none is sufficient alone. Consider: relaxing zoning (HM19) lowers house prices and rents. Does this reduce the LTV premium that drives intergenerational congestion (GP)? Or does the influx of old workers with housing wealth *worsen* the crowding-out of the young? The answer depends on which margin dominates — and no existing model combines all three.

# Readings for This Lecture

## Core papers:

- Guaitoli, G., Pancrazi, R., & Raimondo, D. (2026). *Inter-generational Congestion and Spatial Labour Market Outcomes*. Warwick WP.
- Fajgelbaum, P. & Gaubert, C. (2020). Optimal Spatial Policies, Geography, and Sorting. *QJE*, 135(2), 959–1036.

## Background:

- Lhuillier (2024). Skill accumulation in cities. [GP's Eq. (3)]
- Diamond (2016). Workers' diverging location choices by skill. *AER*. [FG spillovers]
- Glaeser & Gottlieb (2008); Kline & Moretti (2014). [Prior view FG overturns]
- Greaney, Parkhomenko & Van Nieuwerburgh (2025). DUE. [L9; method inherited by GP]

*Thank You!*