Quantitative Spatial Economics

The Allen–Arkolakis Model Calibrated to Italy

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Data: Italian NUTS-2 Regions

20 regions, approximate 2019 values (ISTAT / Eurostat / INPS)

Key variables:

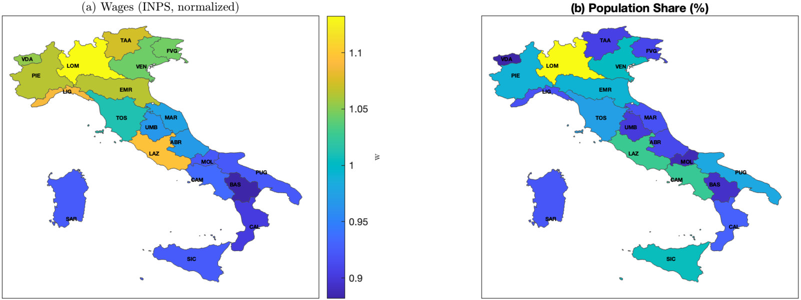
- Population L_j (total $\approx 60.4\,\text{M}$)
- Average annual wage w_j (INPS administrative records)
- GDP per capita (initial productivity proxy)
- Geographic centroids (for bilateral distances)

North–South gradient:

- Northern wages: 30.8–33.5 k €
- Southern wages: 26.1–28.6 k €
- Wage ratio $\approx 1.2 : 1$
- GDP/cap ratio $\approx 2 : 1$

Data Maps

Figure 1: Data -- Wages and Population Shares



Spatial Equilibrium

Geography: $\{\tau_{ij}, \bar{A}_i, \bar{u}_i\}_{i,j\in\mathcal{N}}$ **Parameters**: $\{\sigma, \alpha, \beta\}$

Combining goods market clearing ($w_i L_i = \sum_j X_{ij}$) with free mobility ($w_i u_i / P_i = W$) and substituting CES prices, trade shares, agglomeration ($A_i = \bar{A}_i L_i^{\alpha}$), and congestion ($u_i = \bar{u}_i L_i^{\beta}$):

Equilibrium conditions (AA2014, eqs. 31–32)

$$W^{\sigma-1} w_i^{\sigma} L_i^{1-\alpha(\sigma-1)} = \sum_j \left(\bar{A}_i \bar{u}_j / \tau_{ij}\right)^{\sigma-1} w_j^{\sigma} L_j^{1+\beta(\sigma-1)} \tag{31}$$

$$W^{\sigma-1} w_i^{1-\sigma} L_i^{\beta(1-\sigma)} = \sum_j \left(\bar{u}_i \bar{A}_j / \tau_{ji}\right)^{\sigma-1} w_j^{1-\sigma} L_j^{\alpha(\sigma-1)} \tag{32}$$

(31) — *Revenue*: region i's wage bill = sum of what i earns selling to all j
(32) — *Expenditure*: region i's spending power = sum of what i spends importing from all j

Plus labor clearing: $\sum_i L_i = \bar{L} \implies 2N+1$ equations, $2N+1$ unknowns ($\{w_i, L_i\}, W$)

Solution Algorithm

Iterative fixed point on equations (31)–(32)

Step 0 — Initial guess: start from $\{w_i^{(0)}, L_i^{(0)}\}$ (e.g. data wages and population shares)

At each iteration t, evaluate the RHS of (31) and (32) using current $\{w_i^{(t)}, L_i^{(t)}\}$:

Step 1 — Ratio trick (divide (31) by (32)):

- $W^{\sigma-1}$ cancels $\Rightarrow$ no need to track welfare explicitly
- Collect w_i terms on the LHS: exponent becomes $2\sigma - 1$

$$w_i^{\text{new}} = \left(\frac{\text{RHS of (31)}}{\text{RHS of (32)}} \cdot L_i^{-\gamma_L} \right)^{1/\gamma_w}, \qquad \gamma_L = 1 + (\beta - \alpha)(\sigma - 1), \;\; \gamma_w = 2\sigma - 1$$

Step 2 — Population update (use (31) alone, given w_i^{new}):

$$Z_i = \left(\frac{\text{RHS of (31)}}{(w_i^{\text{new}})^{\sigma}} \right)^{1/(1-\alpha(\sigma-1))}, \qquad L_i^{\text{new}} = \frac{Z_i}{\sum_k Z_k} \bar{L}$$

Step 3 — Damping: $\log x^{t+1} = (1-d) \log x^t + d \log x^{\text{new}}, \quad d = 0.1$

Converge when $\max_i(|\Delta \log L_i|, |\Delta \log w_i|) < 10^{-10}$

Double Model Inversion

Problem: $\bar{A}_j$ and $\bar{u}_j$ are unobserved (2N unknowns, 2N data targets)

Outer loop embeds the solver; at each iteration k:

1. Solve full GE with current $(\bar{A}^{(k)}, \bar{u}^{(k)}) \longrightarrow (L^{(k)}, w^{(k)})$
2. Log-linear corrections:

$$\log \bar{u}_j^{(k+1)} = \log \bar{u}_j^{(k)} + \gamma_u \cdot \log \left(\frac{L_j^{\text{data}}}{L_j^{(k)}} \right)$$

$$\log \bar{A}_j^{(k+1)} = \log \bar{A}_j^{(k)} + \gamma_A \cdot \log \left(\frac{w_j^{\text{data}}}{w_j^{(k)}} \right)$$

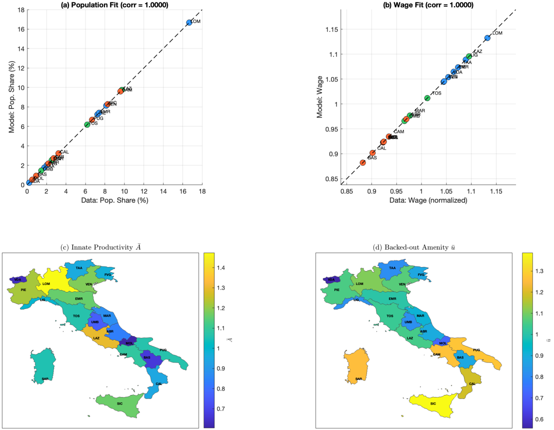
Intuition

Region too small $\Rightarrow$ raise amenity. Wage too low $\Rightarrow$ raise productivity.

Damping: $\gamma_u = 0.3$, $\gamma_A = 0.2$ (productivity more cautious — stronger cross-effects through trade).

Baseline Equilibrium

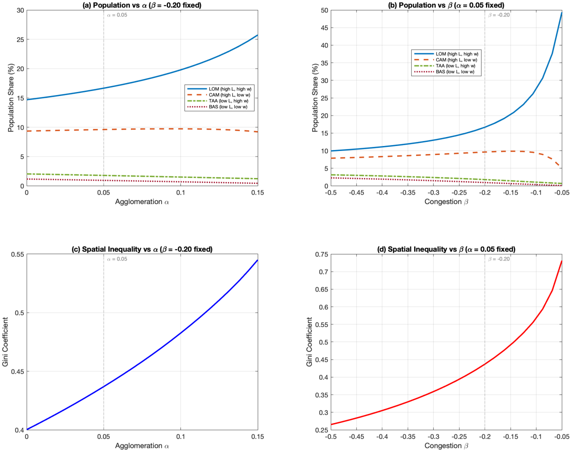
Figure 2: Baseline Equilibrium



(a,b) Perfect fit by construction. (c) Innate $\bar{A}_j$: North–South gradient. (d) $\bar{u}_j$: *opposite* — South has high amenities (rationalizes population despite low wages).

Agglomeration and Congestion

Figure 4: Comparative Statics -- Agglomeration and Congestion

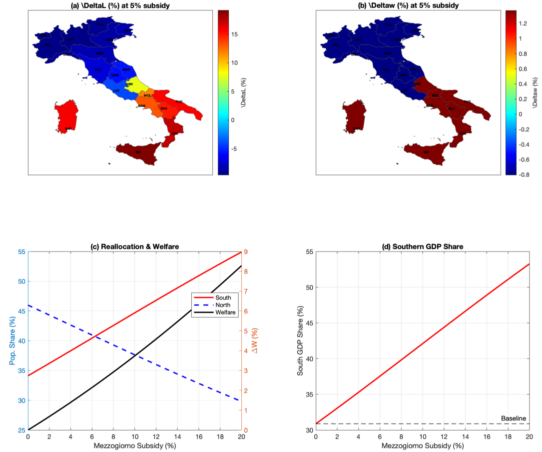


Lombardia (high L, high w), **Campania** (high L, low w), **Trentino** (low L, high w), **Basilicata** (low L, low w). Higher $\alpha \Rightarrow$ concentration; stronger $|\beta| \Rightarrow$ dispersion.

Mezzogiorno Productivity Subsidy

Experiment: raise $\bar{A}_j$ in 8 Southern regions by 0–20%

Figure 5: Mezzogiorno Productivity Subsidy



Maps at 5% subsidy. Welfare rises monotonically — corrects inefficiency (South “too small”).

High-Speed Rail Corridor

Experiment: reduce τ_{ij} along Milano–Roma–Napoli (0–60%)

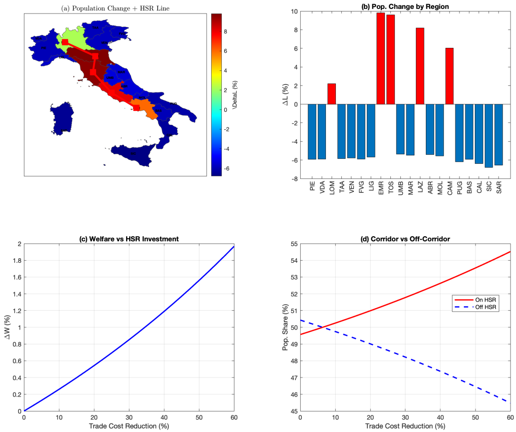


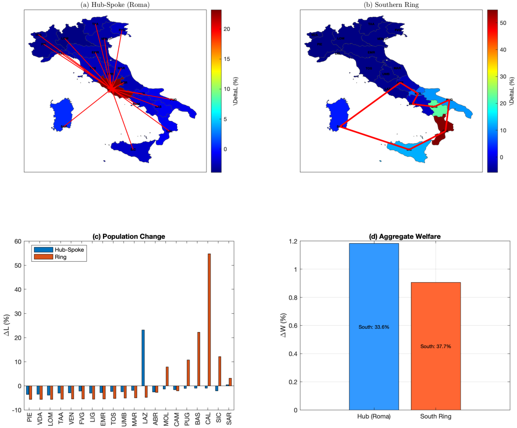
Figure 6: High-Speed Rail Corridor (Milano - Roma - Napoli)

Corridor regions gain; peripheral regions may lose. Linear infrastructure creates spatial winners and losers.

Hub-and-Spoke vs. Southern Ring

Equal-budget comparison: Roma hub (-25% all routes) vs. Southern ring

Figure 7: Infrastructure -- Hub-Spoke (Roma) vs Southern Ring (base South share: 34.1%)



Hub-spoke: higher aggregate ΔW ($+1.21\%$). Ring: larger Southern gains. Classic **equity–efficiency tradeoff**.