Online Appendix for "The Heterogeneous Great Moderation"

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Abstract

The online Appendix reports the empirical rejection frequencies for the GMM-based time domain counterpart of the SCI tests developed in the main text, as well as the results of applying the SCI tests to disaggregated data of Consumption and Investment.

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A Appendix: GMM

In this section I discuss an alternative approach for testing a break in the variance and covariance of a series at particular frequencies using a GMM approach. I show that the small sample proprieties of this approach are worse than for the SCI tests presented in the main paper.

The GMM approach requires the following steps: first, the series of interest should be filtered at a particular interval of frequencies using a band-pass filter. Then, their variance, or covariance, and their standard errors are computed using a GMM estimator: note that in order to calculate the optimal weighing matrix with the Newey and West (1994) procedure, a bandwidth and a smoothing window must be selected. Finally, the time domain equivalent of the SCI test, namely the Average LM test (ALM), the Exponential LM test, or the Nyblom test (NYB), can be directly applied to test whether these parameters have experienced a structural break at an unknown date.[1]

In this section the data generating process is the factor-hoarding model proposed by Burnside and Eichenbaum (1996), a simpler model than the DSGE model presented in the main version. First, Table 1, Table 2, and Table 3, report the empirical rejection frequencies for the SCI tests, respectively for $T = 250$, $T = 500$, and $T = 1000$. The empirical rejection frequencies for the GMM tests are presented in Table 4, Table 5, and Table 6, respectively for $T = 250$, $T = 500$, and $T = 1000$. I use a Bartlett window and its corresponding optimal bandwidth. In particular Newey and West (1994) shows that asymptotically the optimal bandwidth for this window is given by:

$$b = \left[4 \left(\frac{T}{100} \right)^{\frac{2}{9}} \right].$$

As the Tables below display, the GMM approach performs considerably worse than the SCI test in the small sample, since its empirical rejection frequencies are far from their nominal values for the three sample sizes considered, both at Business Cycle and Medium-Cycle frequencies, and for any of the sample sizes considered.

This result should not be surprising. Haan and Levin (1996), and Kiefer et al. (2000) have discussed the unsatisfactory small sample proprieties of GMM estimators, related in particular to the choice of the bandwidth. In fact, whereas the Bartlett windows have been shown to have satisfactory properties, the choice of the bandwidth is a problematic issue. In fact, only asymptotic results related to the optimal rate of convergence of a bandwidth have been proposed in the literature, whereas there are no similar guidelines for the small sample

[1] See Nyblom (1989), Andrews (1993), and Andrews et al. (1996).

problem. Second, the choice of the bandwidth implicitly implies a trade-off between the bias of the estimator and its variance. Therefore, the choice of the bandwidth in a small sample is not a trivial concern in practice, and with my calculation I show that although the choice of the bandwidth has been conducted considering an asymptotic optimal rule, the imprecision of the test statistics is evident. Similar results are obtained if the choice of the bandwidth is guided by the Andrews (1991)'s procedure.

On the other hand, the Spectral Covariance Instability tests do not suffer from the same problem. In fact, as stated in Priestley (1981), the Integrated Cospectrum does not require any choice of a bandwidth. In fact, the Integrated Cospectrum is estimated as the integral of the sample periodogram and the integration procedure along the frequencies works directly as a smoothing function. However, since the integration does not require the specification of any bandwidth parameter, the Integrated Cospectrum does not suffer from any trade-off between its bias and its variance of the estimation. This feature depends on a common result in spectral analysis: although the point estimated of the periodogram at a frequency ω, $\hat{I}_N(\omega)$, is not a consistent estimator of the spectral density of the process at that frequency, $h(\omega)$, the integral of the estimated periodogram $\hat{H}(\omega) = \int_{-\pi}^{\omega} \hat{I}_N(\omega)\, d\phi$ is indeed a consistent estimator of the intergrated spectrum $H(\omega) = \int_{-\pi}^{\omega} h(\omega)\, d\phi$[2].

In conclusion, although a GMM approach can be followed to test for a break for the variances and covariances at particular frequencies, this procedure requires a not trivial choice of the bandwidth and has worse small sample properties than the frequency domain approach presented in this paper.

[2] For further details see Priestley (1981), p.471.

Table 1 – Empirical Rejection Frequencies for the SCI test. T=250

		Business Cycle (6-32)			Medium Cycle (6-80)		
Nominal Significance		**0.100**	**0.050**	**0.010**	**0.100**	**0.050**	**0.010**
SAW Test	$H_{1,1}$	0.119	0.069	0.014	0.141	0.079	0.016
	$H_{2,2}$	0.121	0.071	0.014	0.142	0.080	0.016
	$H_{1,2}$	0.120	0.070	0.014	0.141	0.080	0.016
SEW Test	$H_{1,1}$	0.160	0.094	0.035	0.173	0.110	0.041
	$H_{2,2}$	0.163	0.094	0.035	0.171	0.107	0.042
	$H_{1,2}$	0.162	0.094	0.035	0.172	0.108	0.043
SN Test	$H_{1,1}$	0.117	0.059	0.004	0.126	0.065	0.013
	$H_{2,2}$	0.118	0.059	0.005	0.129	0.066	0.013
	$H_{1,2}$	0.118	0.059	0.004	0.129	0.064	0.013

Note: The Table displays the empirical rejection frequencies of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model proposed by Burnside and Eichenbaum (1996). The series have a length of 250 periods. The table is based on 4000 Monte Carlo repetitions.

Table 2 – Empirical Rejection Frequencies for the SCI test. T=500

		Business Cycle (6-32)			Medium Cycle (6-80)		
Nominal Significance		0.100	0.050	0.010	0.100	0.050	0.010
SAW Test	$H_{1,1}$	0.113	0.065	0.012	0.132	0.079	0.015
	$H_{2,2}$	0.113	0.066	0.012	0.134	0.079	0.015
	$H_{1,2}$	0.113	0.066	0.012	0.133	0.079	0.015
SEW Test	$H_{1,1}$	0.138	0.087	0.029	0.154	0.090	0.035
	$H_{2,2}$	0.135	0.085	0.029	0.154	0.091	0.034
	$H_{1,2}$	0.135	0.086	0.029	0.154	0.092	0.035
SN Test	$H_{1,1}$	0.107	0.058	0.013	0.119	0.064	0.012
	$H_{2,2}$	0.111	0.058	0.014	0.122	0.064	0.012
	$H_{1,2}$	0.109	0.058	0.014	0.121	0.064	0.012

Note: The Table displays the empirical rejection frequencies of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model proposed by Burnside and Eichenbaum (1996). The series have a length of 500 periods. The table is based on 4000 Monte Carlo repetitions.

Table 3 – Empirical Rejection Frequencies for the SCI test. T=1000

		Business Cycle (6-32)			Medium Cycle (6-80)		
Nominal Significance		0.100	0.050	0.010	0.100	0.050	0.010
SAW Test	$H_{1,1}$	0.109	0.059	0.010	0.124	0.069	0.014
	$H_{2,2}$	0.109	0.057	0.011	0.124	0.070	0.015
	$H_{1,2}$	0.109	0.058	0.011	0.124	0.069	0.014
SEW Test	$H_{1,1}$	0.125	0.078	0.022	0.144	0.078	0.022
	$H_{2,2}$	0.124	0.077	0.022	0.148	0.080	0.022
	$H_{1,2}$	0.124	0.078	0.022	0.145	0.078	0.022
SN Test	$H_{1,1}$	0.100	0.052	0.008	0.118	0.059	0.012
	$H_{2,2}$	0.102	0.052	0.008	0.117	0.061	0.012
	$H_{1,2}$	0.100	0.052	0.008	0.11	0.060	0.012

Note: The Table displays the empirical rejection frequencies of the SCI test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model proposed by Burnside and Eichenbaum (1996). The series have a length of 1000 periods. The table is based on 4000 Monte Carlo repetitions.

Table 4 – Empirical Rejection Frequencies for the GMM test. T=250

		Business Cycle			Medium Cycle		
Nominal Significance		0.100	0.050	0.010	0.100	0.050	0.010
ALM Test	$H_{1,1}$	0.411	0.329	0.226	0.344	0.273	0.168
	$H_{2,2}$	0.411	0.329	0.227	0.346	0.272	0.169
	$H_{1,2}$	0.410	0.328	0.226	0.345	0.272	0.168
ELM Test	$H_{1,1}$	0.498	0.431	0.354	0.459	0.391	0.308
	$H_{2,2}$	0.498	0.432	0.354	0.459	0.391	0.309
	$H_{1,2}$	0.498	0.431	0.354	0.459	0.391	0.309
NYB Test	$H_{1,1}$	0.382	0.304	0.201	0.324	0.250	0.143
	$H_{2,2}$	0.382	0.305	0.201	0.325	0.250	0.144
	$H_{1,2}$	0.382	0.303	0.201	0.323	0.250	0.144

Note: The table displays the empirical rejection frequencies of the GMM test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The bandwidth is chosen using the Newey and West (1994)'s procedure. The series have a length of 250 periods. The table is based on 4000 Monte Carlo repetitions. The data generating process is the factor hoarding RBC model proposed by Burnside and Eichenbaum (1996).

Table 5 – Empirical Rejection Frequencies for the GMM test. T=500

		Business Cycle			Medium Cycle		
Nominal Significance		0.100	0.050	0.010	0.100	0.050	0.010
ALM Test	$H_{1,1}$	0.293	0.206	0.107	0.235	0.170	0.087
	$H_{2,2}$	0.292	0.207	0.108	0.236	0.168	0.087
	$H_{1,2}$	0.292	0.207	0.108	0.236	0.168	0.087
ELM Test	$H_{1,1}$	0.373	0.310	0.207	0.316	0.251	0.168
	$H_{2,2}$	0.374	0.310	0.207	0.314	0.250	0.167
	$H_{1,2}$	0.373	0.310	0.207	0.315	0.250	0.167
NYB Test	$H_{1,1}$	0.277	0.182	0.096	0.222	0.153	0.077
	$H_{2,2}$	0.277	0.182	0.096	0.221	0.152	0.077
	$H_{1,2}$	0.277	0.182	0.096	0.221	0.152	0.077

Note: The Table displays the empirical rejection frequencies of the GMM test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The bandwidth is chosen using the Newey and West (1994)'s procedure. The series have a length of 500 periods. The table is based on 4000 Monte Carlo repetitions. The data generating process is the factor hoarding RBC model proposed by Burnside and Eichenbaum (1996).

Table 6 – Empirical Rejection Frequencies for the GMM test. T=1000

		Business Cycle			Medium Cycle		
Nominal Significance		0.100	0.050	0.010	0.100	0.050	0.010
ALM Test	$H_{1,1}$	0.227	0.157	0.072	0.188	0.120	0.057
	$H_{2,2}$	0.227	0.157	0.072	0.185	0.120	0.057
	$H_{1,2}$	0.227	0.157	0.072	0.186	0.120	0.057
ELM Test	$H_{1,1}$	0.287	0.211	0.131	0.236	0.169	0.100
	$H_{2,2}$	0.287	0.210	0.131	0.235	0.170	0.097
	$H_{1,2}$	0.287	0.210	0.130	0.235	0.169	0.098
NYB Test	$H_{1,1}$	0.214	0.144	0.064	0.177	0.110	0.049
	$H_{2,2}$	0.215	0.143	0.064	0.176	0.110	0.049
	$H_{1,2}$	0.214	0.143	0.064	0.176	0.110	0.049

Note: The Table displays the empirical rejection frequencies of the GMM test for testing the presence of a structural break of the variance and covariance of output and investment, generated with the factor-hoarding model described in the appendix. The bandwidth is chosen using the Newey and West (1994)'s procedure. The series have a length of 1000 periods. The table is based on 4000 Monte Carlo repetitions. The data generating process is the factor hoarding RBC model proposed by Burnside and Eichenbaum (1996).

B Appendix: Application of SCI test to disaggregated data

In this section, I study whether the results presented in section 5 of the main paper hold for disaggregated data for consumption and investment. First, I consider the consumption series and its disaggregated components, namely Durable, Non-Durable, and Services.[3] Table 7 displays the same results as described in the paper: at High-Frequencies and at the Higher-Business Cycle component, the tests detect the presence of a significant break, except for the covariance between Durable and Non-Durable consumption at High-Frequencies. However, the break is not present when considering the other two components. We obtain the same regularities for disaggregated component of investment, as displayed in Table 8, with the only exception that the Great Moderation is present only at High-Frequencies for the Non-Residential investment.

[3]Source: BEA.

Table 7 – p-values for the SCI test (Consumption Disaggregated Data)

		High-Freq. [2=6]				Higher-Bus. Cycle [6-16]			
		Cons	Dur	Non-Dur	Serv	Cons	Dur	Non-Dur	Serv
SAW Test	Cons	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.11
	Dur		0.00	0.53	0.00		0.00	0.03	0.00
	Non-Dur			0.00	0.00			0.03	0.00
	Serv				0.00				0.11
SEW Test	Cons	0.00	0.00	0.00	0.16	0.00	0.00	0.00	0.04
	Dur		0.00	0.51	0.00		0.00	0.04	0.00
	Non-Dur			0.00	0.00			0.03	0.00
	Serv				0.00				0.13
SN Test	Cons	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.14
	Dur		0.00	0.53	0.00		0.00	0.03	0.00
	Non-Dur			0.00	0.00			0.02	0.00
	Serv				0.00				0.10
		Lower-Bus. Cycle [16-32]				Medium-Freq. [32-80]			
		Cons	Dur	Non-Du	Serv	Cons	Dur	Non-Du	Serv
SAW Test	Cons	0.15	0.11	0.32	0.23	0.45	0.44	0.46	0.18
	Dur		0.15	0.06	0.03		0.49	0.72	0.28
	Non-Du			1.00	0.09			0.89	0.25
	Serv				0.19				0.19
SEW Test	Cons	0.12	0.06	0.13	0.26	0.40	0.32	0.32	0.19
	Dur		0.09	0.04	0.04		0.38	0.67	0.26
	Non-Du			1.00	0.09			0.87	0.32
	Serv				0.23				0.20
SN Test	Cons	0.17	0.13	0.34	0.23	0.53	0.54	0.56	0.17
	Dur		0.17	0.08	0.35		0.57	0.79	0.24
	Non-Du			1.00	0.27			1.00	0.08
	Serv				0.21				0.10

Note: The Table displays the p-values for testing the presence of a structural break on the variance and covariance of consumption and its disaggregated components (Durable, Non-Durable and Services). The sample is 1947:1-2007:4. The variables are defined in real-per capita terms, and are obtained from NIPA.

Table 8 – p-values for the SCI test (Investment Disaggregated Data)

		High-Freq [2-6]			Higher-Business Cycle [6-16]		
		Inv	Resid	Non-Resid	Inv	Resid	Non-Resid
SAW Test	Inv	0.00	0.00	0.12	0.00	0.02	0.00
	Resid		0.00	0.00		0.03	0.07
	Non-Resid			0.02			0.02
SEW Test	Inv	0.00	0.00	0.10	0.00	0.02	0.00
	Resid		0.00	0.00		0.04	0.07
	Non-Resid			0.00			0.02
SN Test	Inv	0.00	0.00	0.13	0.00	0.02	0.00
	Resid		0.00	0.00		0.04	0.07
	Non-Resid			0.02			0.13

		Lower-Busi Cycle [16-32]			Medium-Freq.[32-80]		
		Inv	Resid	Non-Resid	Inv	Resid	Non-Resid
SAW Test	Inv	0.36	0.39	0.11	1.00	1.00	1.00
	Resid		0.71	0.06		1.00	1.00
	Non-Resid			0.13			1.00
SEW Test	Inv	0.40	0.41	0.15	1.00	1.00	1.00
	Resid		0.73	0.06		1.00	1.00
	Non-Resid			0.16			1.00
SN Test	Inv	0.32	0.37	0.22	1.00	1.00	1.00
	Resid		0.67	0.07		1.00	1.00
	Non-Resid			0.13			1.00

Note: The Table displays the p-values for the testing the presence of a structural break on the variance and covariance of investment and its disaggregated components (Residential and Non-Residential Investments). The sample is 1947:1-2007:4. The variables are defined in real-per capita terms, and are obtained from NIPA

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