

Natural Expectations and Home Equity Extraction^{*}

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Abstract

In this paper we propose a novel explanation for the increase in households' leverage during the U.S. housing boom in the early 2000s. Specifically, we apply the theory of natural expectations, proposed by [Fuster et al. \(2010\)](#), to show that biased expectations on the two sides of the credit market have been a key determinant of the surge in households' leverage but also that inaccurate long-run expectations on behalf of financial intermediaries are a necessary - yet so far overlooked - ingredient for matching the observed debt and interest rates dynamics.

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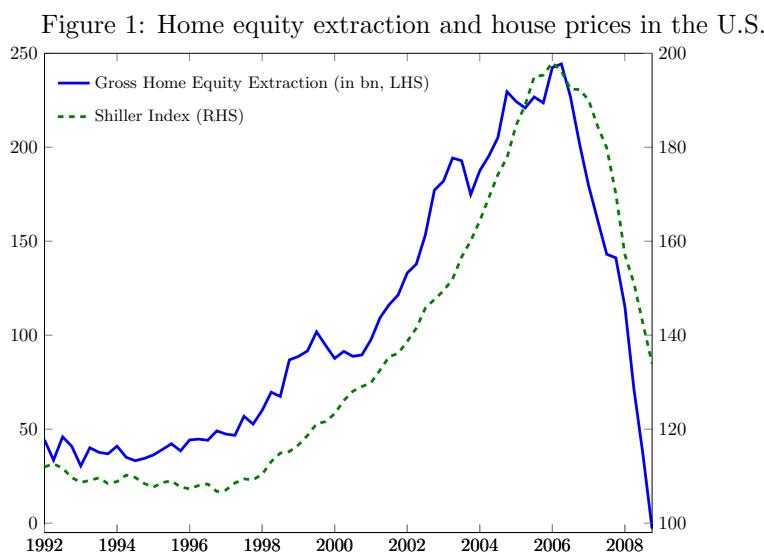
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1 Introduction

From 1999 to the end of 2006, U.S. household debt relative to income grew sharply, from 64 percent to more than 100 percent.¹ A strong appreciation in housing prices accompanied the increase in debt: the Standard & Poor’s Case-Shiller Home Price Index soared by 65 per cent in real terms in the same time span. Unlike previous episodes of heated housing markets, this housing price boom has been characterized by a surge in households’ home equity extraction (HEE), through cash-out refinancing of mortgages, second lien home equity loans, or home equity lines of credit (HELOCs). In 1992 the value of HEE was about \$41 billion (in 2006 dollars); at the end of 1999 it more than doubled to about \$95 billion; and from 2000 to 2006, when housing price growth was at its peak, HEE almost tripled (Figure 1).² Also, Greenspan and Kennedy (2005) document that households’ gross home equity extraction as a fraction of disposable income increased from less than 3 percent to about 10 percent between 1997 and 2005.³



Note: This figure displays the flows of home equity extraction (solid blue line, left scale) in the U.S. in billions of dollars along with the Shiller’ Real Home Price Index (dashed green line, right scale). Home equity extraction is computed as a four quarters moving average of Gross Equity Extraction divided by the Consumer Price Index. The series, computed according to the methodology in Greenspan and Kennedy (2005), is available at <http://www.calculatedriskblog.com/2009/03/q4-mortgage-equity-extraction-strongly.html> (retrieved 7 August 2014). The Real Home Price Index is available at the Robert Shiller’s website (<http://www.econ.yale.edu/~shiller/data.htm>, retrieved 7 August 2014).

One of the explanations provided in the literature for the large increase in household debt during the house price boom is related to the inability of households to make sound financial decisions when exposed to housing-related financial products. The explanations for such behavior range from money illusion (Brunnermeier and Julliard 2008), to limited financial literacy (Gerardi et al. 2010 and Lusardi and Tufano 2015), and to inability to accurately forecast house prices (Goodman and Ittner 1992). However, while the literature has focused on the behavior of agents on the demand side of the debt market, little has been said about the role of agents on the supply-side.

¹Source: US. Bureau of Economic Analysis (GDP, BEA Account Code: A191RC1) and Federal Reserve System, Flow of Funds (Households and nonprofit organizations; total mortgages; liability, id: Z1/Z1/FL153165005.Q).

²Source: Federal Reserve System, Flow of Funds.

³Other works that have examined the role of home equity-based borrowing include Mian and Sufi (2011), Disney and Gathergood (2011), and Brown et al. (2013), among others.

In this paper we propose a framework in which (i) the availability of financial instruments allows agents to borrow today against the future expected value of their houses; and (ii) both households and financial intermediaries can be subject to inaccurate long-run house price expectations. We show that biased expectations on the two sides of the housing-related credit market adequately explained the surge in households’ leverage during the 2000s housing price boom, but also that, more importantly, inaccurate long-run expectations on behalf of financial intermediaries are a necessary - yet so far overlooked - ingredient for matching the observed debt and interest rates dynamics. This finding is consistent with the evidence provided in [Kaplan et al. \(2017\)](#) and in [Bailey et al. \(2017\)](#), which, using a different approach than ours to capture optimism, show that the increased leverage observed in the early 2000s relied on lenders’ optimism.

Our framework is based on the concept of *natural expectations*, first described in [Fuster et al. \(2010\)](#) and [Fuster et al. \(2012\)](#). These papers build on an asset-pricing setting in which: (1) fundamentals of the economy are hump-shaped, exhibiting momentum in the short run and partial mean reversion in the long run, which, however, is hard to identify in small samples; and (2) agents do not know that fundamentals are hump-shaped; instead, base their beliefs on too parsimonious models that fit the available data, thus generating inaccurate long-run predictions. We apply a similar approach to the housing-credit market, assuming that our economy’s homeowners take housing prices as given; they derive long-run house price forecasts in order to quantify their future housing wealth and to decide how much equity to extract. Similarly, financial intermediaries need to forecast future house prices to choose the supply of home equity loans.

The assumption that households behave in line with the *natural expectations* theory when confronting house prices is largely supported by empirical work. For example, [Goodman and Ittner \(1992\)](#) surveys the early literature about the optimism of homeowners in assessing the future values of their homes and documents that households overestimate home prices by 4 to 16 percent. Using survey data in the period 2002-2012, [Case et al. \(2012\)](#) find that households’ forecasts under-predicted actual realizations in the short-run (one year) but were “abnormally high” in the long run (10 years). Similar evidence has been documented in [Shiller \(2007\)](#) and [Benitez-Silva et al. \(2008\)](#). A further strand of literature has also highlighted the interaction between optimistic forecasts and households’ borrowing, which may give rise to belief-driven housing cycles ([Kuang 2014](#)).

Nevertheless, households are only one side of the housing-related debt market. In fact, financial institutions supply credit to households and, if they did not share the same optimistic forecasts, they would be reluctant to provide home equity loans at low interest rates. Hence, a key contribution of this paper is to show that in order to replicate the dynamics of boom-bust episodes like the one recently observed in the U.S., one needs also *natural expectations* on behalf of financial intermediaries, in the sense that they, too, had to ignore any form of long-run mean reversion in housing prices after the positive and strong short-run momentum.⁴ Seen from a different perspective, the main claim of the paper is that in order for optimistic beliefs to be the only explanation for the

⁴In this respect our paper differs from the related work by [Glaeser and Nathanson \(2015\)](#). Indeed, while both papers rely on models that are able to generate house price beliefs with hump shaped dynamics, our paper focuses more on the effects of housing beliefs on the debt market and on the relative contribution of households’ and financial intermediaries’ beliefs in explaining house related debt dynamics.

large increase in household leverage, they need to be shared by both households and banks.

Treating financial institutions as *natural* agents is coherent with other studies about the behavior of housing market experts during the boom phase. More precisely, Foote et al. (2012) show that industry analysts and economists were on average optimistic about housing price dynamics and that such beliefs were not without consequence, as those who originated and securitized mortgages incurred in significant losses and even “the executives most likely to understand the subprime-lending process had made personal investment decisions that exposed them to subprime risk” (p. 19). Such a finding is also in Cheng et al. (2014), where the authors show that, during the boom, securitization investors and issuers increased their housing exposure, thus revealing that they were not aware of a bust phase coming. Interestingly, the authors also find that those experts perceived the increase in their incomes, which were intrinsically related to the dynamics of the housing market as permanent. Finally, Barberis (2013) points at the adoption of poor statistical models by financial intermediaries as one of the possible causes of the excessive lending recorded during the house price boom. We take stock of this literature and investigate the effects of optimistic beliefs on financial intermediaries. At the same time, we do not attempt at investigating where these optimistic beliefs are originated from: they may indeed arise from bounded rationality or psychological biases of bank managers as well as from short-sighted incentives or from the actual use of bad forecasting models. From our point of view, these explanations are observationally equivalent as they all lead agents on the supply side of the credit market not to adequately take into account the long-run behavior of the statistical objects they form their expectations upon. When applied to banks, the theory of *natural* expectations provides an appealing microfoundation to the increased supply of credit during the boom phase, which has been documented in reduced form in Justiniano et al. (2014) via what the authors label as an easing of the *lending constraint*. While we acknowledge that competing explanations can be brought forward to explain the surge in home equity extraction in the US housing market at the beginning of the century,⁵ nonetheless, we offer a simple and coherent framework of *natural* expectations on both sides of the debt market that replicates fairly accurately the bubbly dynamics observed in the US housing market.

The first step in the paper consists then in showing that housing prices are characterized by hump-shaped dynamics, which imply a large momentum in the short run and partial mean reversion in the long run. Thus, we compare four different statistical models to estimate and forecast housing price dynamics. First, we consider two possible dimensions that lead to *natural* expectations: (1) an inner tendency of agents to incorporate a small set of explanatory variables when estimating a model, in line with the findings in Beshears et al. (2013); and (2) a limited ability of agents to consider a large set of data when estimating the model, in line with the assumption of extrapolative expectations applied to the housing market.⁶ We compare the performance of these models with the ones of two rigorous and more sophisticated statistical approaches to modeling and forecasting housing prices, which differ in the information criterion used to select the most appropriate specification. We find

⁵As a way of example, Hurst and Stafford (2004) and Chen et al. (2013) suggest that such behavior is coherent with a precautionary saving motive of liquidity constrained households.

⁶See Goetzmann et al. (2012), Abraham and Hendershott (1994), Muellbauer and Murphy (1997) and Piazzesi and Schneider (2009).

that models that incorporate hump-shaped dynamics are not preferred, in terms of in-sample fit, to more parsimonious models that ignore long-run mean reversion. As a result, the use of simple models leading to natural expectations is fully justifiable in terms of in-sample performance. However, we demonstrate that models that have diverse degrees of ability to capture hump-shaped dynamics in housing prices, while leading to comparable short-run predictions, may generate a wide range of long-run forecasts. Hence, from an in-sample fit perspective, it is legitimate for agents to make use of relatively simple models; the drawback however is that in this way they fail to take into account the partial mean reversion of housing prices in the long run.⁷

The second contribution of the paper is to link long-run housing price forecasts to the optimal behavior of agents in the credit market. We therefore introduce a tractable model of a collateralized credit market populated by a representative household and a bank. The household can obtain credit from the bank by pledging its house as collateral.⁸ In each period, the household decides how much to consume and how much to borrow and, given the realization of the stochastic exogenous housing price, whether to repay its debt or to default and lose the ownership of the house. The amount of debt demanded crucially depends on the expected realizations of the housing price. The bank borrows resources at a prime rate and lends them to the household charging a margin. The bank gains either from debt repayment, in the case of no default from the household, or from the sale of the housing stock, in the case of default. Obviously, the banks' expected future house price is a key determinant of its supply of credit.

In our quantitative assessment, we are mainly interested in examining the extent to which the equilibrium level of debt and its price vary with the ability of agents to take into account possible long run mean-reverting dynamics of housing prices. Hence, we select a housing price path in our model that matches the observed dynamics of the aggregate U.S. housing price in the period 2001-2010, and we vary the specification of the process the agents use to predict future house prices. We consider a large set of specifications (fifty) that are identical in terms of the short-run (one-year ahead) forecast, and in terms of magnitude of the unconditional variance of the housing price process, but that differ in terms of the long-run expectations. Hence, we can rank the different specifications according to their degree of *naturalness*: more *natural* processes ignore the long-run mean reversion of housing prices and predict a higher long-run price; less *natural* processes incorporate a certain degree of housing price adjustment after the short-run momentum and predict a lower long-run price. We obtain four results. First, the theoretical model predicts a positive relationship between the average equilibrium level of debt in the economy in the boom phase and the degree of *naturalness* of agents. Intuitively, after observing an increase in the house price, a more *natural* agent (either a household or a bank) expects a long-lasting housing price appreciation, which gives her strong incentives to demand/supply debt. Second, long-run expectations play a large role from a quantitative point of view: when the economy is populated by more *natural* agents, the debt-to-income ratio during a boom phase is about 55 percent; when the economy is populated by less *natural* agents it falls to

⁷As discussed in Fuster et al. (2010): “there are several reasons that justify the use of simple models: they are easy to understand, easy to explain, and easy to employ; simplicity also reduces the risks of over-fitting”.

⁸The model is related to Cocco (2005), Yao (2005), Li and Yao (2007), Campbell and Cocco (2015), and Brueckner et al. (2012).

35 percent. Recall that the difference in these quantities is solely due to the contrasting long-run expectations on housing prices, since by construction agents have the same short-run expectations in each of the fifty specifications. Third, using data on Gross Home Equity Extraction as computed in Greenspan and Kennedy (2005), we show that the simulated process that better fits the observed debt dynamics during the 2000-2009 episode is characterized by a rather high degree of *naturalness*. Finally, we show that *naturalness* on the supply-side is particularly relevant for explaining the surge in leverage and for the interest rate reduction on home equity loans observed during the housing price boom. In fact, by conducting simple experiments where only the bank or the household (or both) are natural, we highlight that banks' *naturalness* has a larger effect than that of households on the equilibrium level of debt in the economy.

The rest of the paper is organized as follows. In section 2 we discuss the properties of *natural* expectations and their implications for long-run housing price forecasts. In section 3 we describe the theoretical model, and in section 4 we describe its calibration. In section 5 we discuss the quantitative results of the model. Section 6 concludes and summarizes the main findings.

2 Natural House Price Expectations

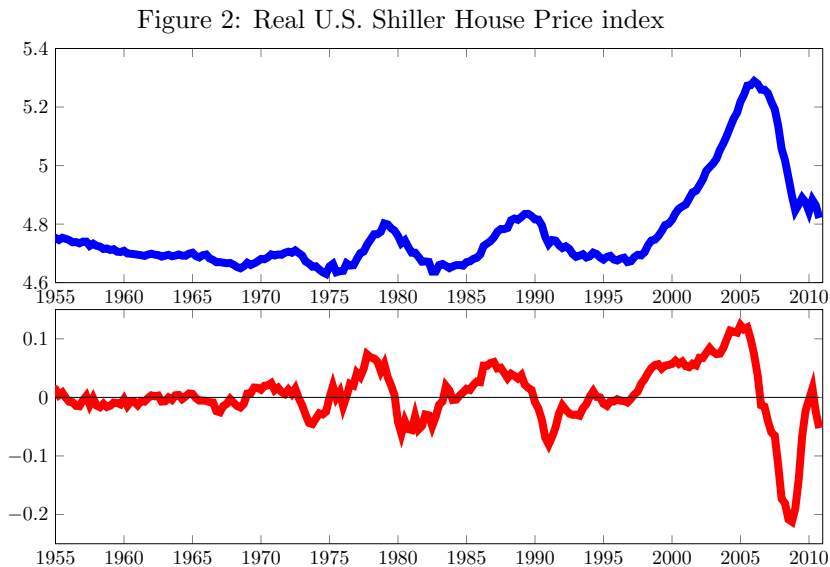
In this section we show three results that justify the use by economic agents of natural expectations with respect to housing prices. First, we show that the time series for the U.S. aggregate housing price is characterized by hump-shaped dynamics, which imply momentum in the short run and partial mean reversion in the long run. Second, we document that models that incorporate hump-shaped dynamics are not preferred, in terms of in-sample fit, to more parsimonious models that ignore long-run mean reversion. As a result, the use of simple models leading to natural beliefs is perfectly justifiable in terms of in-sample performance. Third, we demonstrate that, nevertheless, forecasts based on models with various degrees of ability in capturing the hump-shaped dynamics of housing prices differ over long-run horizons but not in the short-run. Hence, if agents use simple models (for a wide range of good reasons⁹), they fail to forecast the partial mean reversion in housing prices over the long run. Following Fuster et al. (2010), we call the resulting beliefs of these agents *natural expectations*.

2.1 Modeling Natural Expectations for Housing Prices

We start by examining data on the aggregate real U.S. housing price index to see how different modeling approaches vary in their ability to capture hump-shaped long-run dynamics. The series of interest is the quarterly Standard & Poor's Case-Shiller Home Price Index for U.S. real housing prices in the sample 1953:1-2010:4. We start from 1953 as earlier data are only available at annual frequency; we end our analysis in 2010 as we are mostly interested in the "boom gone bust" episode that started in the mid-nineties, peaked in 2005-2006, and then displayed negative growth rates

⁹As Fuster et al. (2010) put: "simple models are easier to understand, easier to explain, and easier to employ; simplicity also reduces the risks of overfitting. Whatever the mix of reasons -pragmatic, behavioral, and statistical-economic agents usually do use simple models to understand economic dynamics".

since the end of 2009. The logarithm of the raw series is plotted in the upper panel of Figure 2. The series displays at least four episodes of boom and bust: the first one in the early '70s, the second one later in the decade, the third one in the '80s, and, finally, the most recent and significant from 1997 to 2005.



Note: This figure plots the Standard & Poor’s Case-Shiller Home Price Index U.S. real housing price index in its level (upper panel) and growth rate (lower panel).

The series is statistically characterized by the presence of a unit root.¹⁰ We therefore consider as a variable of interest its yearly growth rate, displayed in the bottom panel of Figure 2. Notice also that the growth rate of housing prices is characterized by relatively long periods positive growth followed by abrupt declines, which indicate the presence of a rich autocorrelation structure.

We then assume that the process for housing price growth rate, g_t , is autoregressive,¹¹ i.e.:

$$(1 - \Theta^p(L))g_t = \mu + \varepsilon_t, \quad (1)$$

where $\Theta^p(L)$ is a lag polynomial of order p , μ is a constant, and ε_t are *iid* innovations.

We assume that an agent could estimate the model in equation (1) using four different criteria that gather a spectrum of different approaches to estimation and forecasting. Initially, we propose two simple models that capture *natural expectations* on housing prices. Recall that, as in [Fuster](#)

¹⁰To formally test the null hypothesis of presence of a unit root in the house price level, we run the [Phillips and Perron \(1988\)](#) unit root test. We allowed the regression to incorporate from 1 to 15 lags. For any of these specifications the test could not reject the null hypothesis of the presence of a unit root. To check whether the presence of a unit root is driven by the 1997-2007 price boom, we run the test for the shorter sample 1953:1-1996:4. Also in this case, the Phillips-Perron test could not reject the null hypothesis at a 5 percent significance level for any model specifications. In addition, there is no statistical evidence that the house price of growth rate contains unit roots.

¹¹Our modeling choice is justified by [Crawford and Fratantoni \(2003\)](#) who show that linear (ARMA) models are preferred to non-linear housing price models for out-of-sample forecasts. As a robustness check, we have alternatively assumed that the housing price growth rate g_t is an ARMA process of the form $(1 - \Theta^p(L))g_t = \mu + (1 + \Phi^q(L))\varepsilon_t$, where $\Phi^q(L)$ is a lag polynomial of order q . The BIC chooses an ARMA(1,4), whereas the AIC chooses an ARMA(18,5). The impulse response functions are very similar to the one reported in this section when assuming an AR process.

et al. (2010), we define *natural expectations* as the beliefs of agents that fail to incorporate hump-shaped long-run dynamics of the fundamentals. We explore two possible dimensions that lead to *natural expectations*: (1) a limited ability of agents to incorporate a large set of explanatory variables when estimating a model; and (2) a limited ability of agents to consider a large set of data when estimating the model. Regarding the first model, we assume that an agent naively considers a first order polynomial, that is $p = 1$ and $\Theta^p(L) = 1 - \theta_1 L$ when estimating equation (1). This assumption captures behavioral biases, such as a natural attitude to use over-simplified models, as in Beshears et al. (2013) and in Hommes and Zhu (2014). We refer to this model as *intuitive expectations*, consistently with Fuster et al. (2010). Regarding the second model, we assume that an agent has finite memory and accordingly forecasts the model in equation (1) by considering only the most recent observations. In particular, we assume that agents consider only the last $T^{lim} = 100$ observations when estimating the model.¹² The underlying assumption is that agents using this model do not take into account earlier historical housing price dynamics, either because they do not have access to those data, or because they ignore them, or simply because they assign much lower weight to older observations. We refer to this model as *finite memory*.¹³ Notice that the *finite memory* model captures a source of bias that does not emerge because of a possible model misspecification (as for the *intuitive expectations* model), but the bias depends upon the limited amount of information that is relevant for the agent when estimating the model.¹⁴

We then compare the implications of these *natural expectations* models with the ones produced by removing the two above assumptions. In fact, an agent could, to the contrary, make use of all the available observations and/or of more sophisticated econometric techniques to estimate the appropriate lag polynomial in equation (1). When choosing how many parameters to include, a modeler faces a trade-off between improving the in-sample fit of the model and the risk of overfitting the available data, which may result in poor out-of-sample forecasts. Two of the most popular criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). It is not clear which criterion should be preferred by practitioners in small samples.¹⁵ We retain both, considering as third and fourth models the specification of equation (1) obtained when an econometrician uses respectively the AIC criterion and the BIC criterion.

In Table 1 (left panel for the whole sample 1953:1-2010:4) we report point estimates (standard errors in brackets) for four models: $p = 1$, estimated with an *intuitive* model; $p = 6$, estimated with a *finite memory* model; $p = 5$, estimated with the BIC model; $p = 16$, estimated with the AIC model.

¹²We obtain similar results when varying T^{lim} in the range 80-120.

¹³There are other interpretations for this approach. For example, agents might have adopted a “new-era thinking”, which refers to agents deliberately excluding less recent observations because they believe they are not relevant anymore. Alternatively this approach can also capture the assumption of extrapolative expectations in the housing market employed by Goetzmann et al. (2012), Abraham and Hendershott (1994), Muellbauer and Murphy (1997), Piazzesi and Schneider (2009), and it relates to the findings of Agarwal (2007) and Duca and Kumar (2014), which state that younger individuals have statistically significant more propensity to overestimate house prices and to withdraw housing equity, respectively.

¹⁴We assume that the agent with finite memory estimates the model by maximizing information criteria. Since the BIC and AIC select the same length for the lag-polynomial, the two approaches deliver the same results.

¹⁵See McQuarrie and Tsai (1998) and Neath and Cavanaugh (1997) for opposing arguments.

Table 1: Estimation of House Price Growth

p	Whole Sample: 1953:1-2010:4				Subsample: 1953:1-1996:4			
	Intuitive 1	Finite memory 6	BIC 5	AIC 16	Intuitive 1	Finite memory 5	BIC 13	AIC 17
θ_1	0.958*** [0.02]	1.636*** [0.10]	1.330*** [0.06]	1.348*** [0.07]	0.914*** [0.00]	1.129*** [0.10]	1.052*** [0.08]	1.118*** [0.08]
θ_2		-0.581*** [0.19]	-0.221** [0.10]	-0.241*** [0.11]		-0.153 [0.14]	-0.024 [0.11]	-0.136 [0.13]
θ_3		0.100 [0.19]	0.090 [0.10]	0.122 [0.12]		0.219 [0.14]	0.113 [0.11]	0.194 [0.12]
θ_4		-0.789*** [0.18]	-0.614*** [0.10]	-0.841*** [0.12]		-0.540*** [0.14]	-0.695*** [0.11]	-0.805*** [0.12]
θ_5		0.850*** [0.19]	0.355*** [0.06]	0.656*** [0.12]		-0.245*** [0.10]	-0.540*** [0.12]	0.652*** [0.14]
θ_6		-0.259** [0.11]		0.012 [0.13]			0.077 [0.13]	0.004 [0.14]
θ_7				-0.060 [0.13]			-0.073 [0.13]	-0.006 [0.14]
θ_8				-0.457*** [0.13]			-0.459*** [0.13]	-0.562*** [0.14]
θ_9				0.346*** [0.13]			0.425*** [0.12]	0.485*** [0.14]
θ_{10}				0.055 [0.13]			0.049 [0.11]	0.013 [0.14]
θ_{11}				0.121 [0.14]			0.039 [0.11]	0.148 [0.14]
θ_{12}				-0.631*** [0.13]			-0.467*** [0.11]	-0.653*** [0.14]
θ_{13}				0.285** [0.12]			0.218*** [0.08]	0.403*** [0.14]
θ_{14}				0.050 [0.13]				-0.118 [0.12]
θ_{15}				0.136 [0.13]				0.211* [0.12]
θ_{16}				-0.119 [0.08]				-0.291** [0.08]
θ_{17}								0.105 [0.08]

Note: In this table we report the estimates of the autoregressive process in equation (1) when considering four models. The *intuitive* expectations model assumes a first order autoregressive process. The *finite memory* assumes that the agents estimate the model by using only the most recent 100 observations and select the order of the lag polynomial by considering the Bayesian Information Criterion. The BIC and AIC models are estimated by maximizing the two different information criteria when using observation from the whole sample (1953:1-2010:4) (left panel) and in the subsample (1953:1-1996:4) (right panel). The real housing price is the annual growth rate of the Shiller index. Standard errors are in brackets. Significance at 1 percent is indicated by ***, at 5 percent by **, at 10 percent by *.

Notice that there is a remarkable difference in the number of lags selected by the last two models: since the BIC criterion largely penalizes overfitting, it selects much fewer lags than the AIC criterion. Furthermore, the large number of significant parameters for lags greater than one, in particular for the AIC model, confirms that the process of housing price growth has a relatively rich autoregressive structure. Also, notice that most of the statistically significant parameters for lags greater than one enter with a negative sign, a clear indication of hump-shaped dynamics. Consequently, an agent who makes use of a simpler autoregressive model is likely to ignore important dynamics of house price growth. The different long-run implications of the models are summarized by their resulting long-run persistence, as discussed in detail below. Notice that these findings are robust to considering only a more limited sample (1953:1-1996:4) that does not include a recent housing price boom, as reported on the right panel of Table 1.

2.2 In-sample Fit and Long-Run Predictions

In this section we provide evidence that, although drastically contrasting in their underlying assumptions, the above specifications have similar in-sample properties, and they are hardly distinguishable from a statistical point of view. Table 2 reports statistics about the goodness of fit of the four models.

Table 2: In-Sample Fit and Forecasts

	Intuitive ($p = 1$)	finite memory ($p = 6$)	BIC ($p = 5$)	AIC ($p = 16$)
RMSE	0.0148	0.0122	0.0122	0.0113
R^2	0.9130	0.9713	0.9417	0.9531
\bar{R}^2 (adj.)	0.9126	0.9694	0.9404	0.9496
log-likelihood	636.58	682.90	681.14	700.72
p-value LR test (against AR1)			0.13	0.19
One period Ahead Forecast	1.96	2.63	2.33	2.34
Confidence Bands (95%)	[1.90; 1.97]	[2.31; 2.82]	[2.18; 2.44]	[2.18; 2.48]
Long-Run Persistence (LRP)	23.7	24.4	18.7	10.4
Confidence Bands (95%)	[10.3; 31.4]	[6.4; 59.5]	[8.6; 28.9]	[5.1; 17.7]

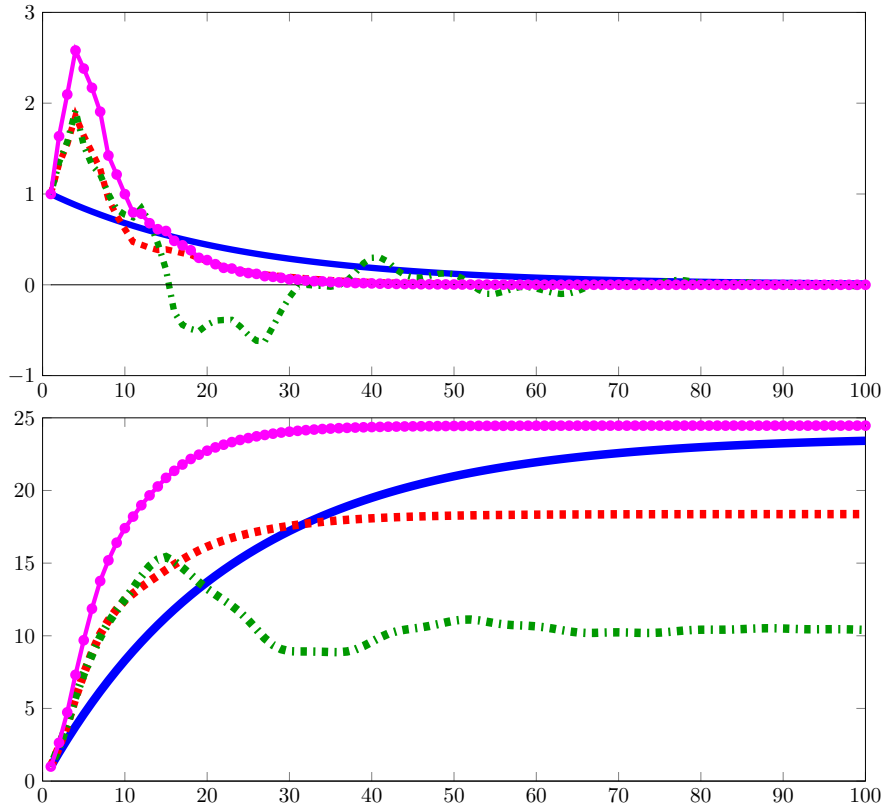
Note: The top panel of this table reports the in-sample fit statistics for the four models for model for housing prices (Intuitive expectations, finite memory model, and for the model selected by the BIC and by AIC). The bottom panel reports statistics regarding the properties of the models about the short-run forecasts and long-run forecasts.

The Root Mean Squared Error (RMSE), the unadjusted coefficient of determination (R^2), and the adjusted coefficient of determination (\bar{R}^2) are very similar across the models.¹⁶ Since the *intuitive* model, the BIC model, and the AIC model are all nested models, we can formally test whether the data can formally reject the null hypothesis that the three models are observationally similar by comparing the log-likelihood evaluated at the unrestricted model parameter estimates and the restricted model parameter estimates. As Table 2 displays, the resulting Likelihood Ratio (LR) test statistics when assuming that the restricted model corresponds to $p = 1$ and the unrestricted model corresponds to $p = 5$ and $p = 16$, respectively, confirm that the models cannot be distinguished on the basis of goodness-of-fit alone. Since the *finite memory* model considers a different sample, it cannot be nested in the other three models. Hence, the LR test cannot be performed. Nevertheless, notice that its likelihood is very similar to the one of the other three models. Notice, too, that the one-quarter-ahead forecasts produced by these models are also similar.

Although the models imply a similar fit to the data and similar short-run predictions, their long-run out-of sample forecast implications are different. We can observe these features of the models by plotting the impulse response functions for a 1 percent positive shock in the housing price growth rate, as displayed in the top panel of Figure 3.

¹⁶Although we do not report them here, the historical in-sample fitted values of the four models are basically indistinguishable. Therefore, they the different empirical models have a very similar ability to capture the in-sample boom-and-bust episodes.

Figure 3: Comparison of Impulse Response Functions



Note: This figure reports the impulse response function (IRF) of housing price growth rate (upper panel) and housing price level (lower panel) to a positive unitary shock. The solid blue line represents the IRF implied by agents that estimate an AR(1) process for the housing price growth rate (intuitive model). The solid-dotted purple line represents the IRF implied by an agents that estimate a process for the housing price growth rate when using only the last 100 observations (finite memory model). The dotted red line represents the IRF for an agent that maximizes the Bayesian Information Criterion and, hence, estimates an AR(5) process for the housing price growth rate. The green dashed line represent the IRF for an agent that maximizes the Akaike Information Criterion and, hence, estimates an AR(16) process for the housing price growth rate.

The *intuitive* model (solid blue line) estimates a very persistent process, as indicated by the value of the parameter of the AR(1) process, equal to 0.96 as reported in Table 1. Consequently, it predicts a long-lasting positive effect of a shock on housing price growth. In contrast, the BIC model (dashed red line) and the AIC model (dotted green line) predict larger short-run responses of housing prices, but they estimate faster reversion after 10-15 quarters. Notice, also, that the practitioner who uses the AIC criterion estimates a negative medium-run response of price-growth after the large boom, but even this model does not particularly succeed of incorporating a large mean reversion component of house price. This fact shows that it is hard to obtain mean-reversion dynamics even with more sophisticated models when estimated in small samples. Finally, the *finite memory* model (dotted purple line) has a very large short-run response and implies a persistence of the positive shock for about 30 quarters, without any sort of mean reversion.

We can obtain insights about the different long-run predictions of the models by plotting the impulse responses of the level of the housing prices, as displayed in the lower panel of Figure 3. These responses are given by the cumulative sum of the impulse responses of the growth rate. An agent using the *finite memory* model (dotted purple line) predicts that, after a positive shock,

housing prices will largely increase for about 25-30 quarters and then stabilize at a high level. An agent using the *intuitive* model (solid blue line) expects a longer persistence of housing price appreciation, which leads to a similar long-run forecasts as with the *finite memory* model. The two more sophisticated models (BIC model, dashed red line, and AIC model, dotted green line) predict a much lower degree of persistence, which leads to lower expected long-run prices. In fact, they prove better in capturing the mean-reversion feature of housing prices than both the *intuitive* model and the *finite memory* model. Notice also, that an econometrician using the AIC criterion expects a depreciation following the initial boom. Furthermore, since the four models are hardly distinguishable in the sample, as pointed out above, it is legitimate to conjecture that these impulse responses are associated with a large degree of uncertainty. Not surprisingly, this is indeed the case, as described in Appendix A.

The long-run dynamics of housing prices are of particular relevance in this paper. A measure of the long-run price estimated after a shock is the long-run persistence of the price level, defined as the long run steady state level after a 1 percent shock. Given that the price level is assumed to follow an ARIMA($p,1,0$) model, the long-run persistence (LRP) can be computed as:

$$LRP = \frac{1}{1 - \sum_{j=1}^p \theta_j}$$

where θ_j , $j = 1, \dots, p$ are the coefficients of the lag polynomial of order p , $\Theta^p(L)$. Table 2 reports the LRP of the processes estimated by the four models as well as their confidence band.

As Table 2 reports, the LRP estimated with an *intuitive* model is larger than the one estimated by agents using a more rigorous statistical approach. In particular, the AR(1) model delivers a long-run persistence that is 30 percent higher than the AR(5) model selected by the BIC, and 80 percent higher than the AR(16) model selected by the AIC.¹⁷ Also, the LRP estimated by the *finite memory* model is similar to the one estimated by the *intuitive* model. This an important result since it shows that agents who use oversimplified models (because of behavioral biases or sample selection) tend to have more optimistic expectations about long-run housing price resulting after a positive shock than agents using more sophisticated models. In Table 6 in Appendix B we report similar results obtained when considering annual data, confirming that our findings are not an artifact of data frequencies.

3 A Model for Home Equity Loans and Natural Expectations

Having shown in the previous sections that entertaining *natural* expectations on housing prices is a legitimate assumption given the rich statistical structure of housing prices time series and

¹⁷As already stated, as a robustness check, we have alternatively assumed that the housing price growth rate g_t is an ARMA process. The BIC and the AIC pick respectively an ARMA(1,4), and an ARMA(18,5). Since the LRP (18.6 for ARMA(1,4) and 12.9 for the ARMA (18,5)) and the Impulse Response functions are very similar to the one estimated with the AR processes we decided to present only the latter.

the in-sample properties of simple statistical models, we now aim at investigating the economic consequences of having agents with *natural* expectations on both sides of the debt market. In this section, therefore, we propose a model in which a representative household and a representative bank interact in a market for home equity loans. Importantly, we allow agents to have a range of expectations upon the evolution of the exogenous housing price that varies with the ability of agents to incorporate long run mean reversion of house prices. Hence, the expectations vary from more *natural* (lower ability to incorporate long-run mean reversion) to less *natural* (greater ability to incorporate long-run mean reversion). Our theoretical model can be used as a laboratory to investigate the extent to which *naturalness* of households and banks has affected the level of debt in the economy during the housing price boom. Also, the model allows for decomposing the relative importance of households' and banks' expectations in the determination of the equilibrium in the market.

3.1 Household

The economy lasts $T < \infty$ periods and is populated by two representative agents: a household and a bank. There are a non-storable consumption good and two assets: housing and debt claims. The household starts at $t = 0$ with an endowment of housing stock h worth p_0h , where p_t denotes the real housing price at time t , and the household is allowed to sell the house only in the final period, at a price p_T , unless it decides to default in any time $t = 1, \dots, T - 1$. In case of default, the household loses the ownership of the house and becomes a renter. Since the household starts with an owned housing stock and with no previous debt, and it does not engage in buying or selling its housing stock, we can interpret the debt claims in the economy as home equity extraction. We assume that the household is endowed in each period with a constant income $y_t = y > 0$. The housing price is an exogenous variable for the agents in our economy.¹⁸

Subject to the repayment of debt accumulated in the past, in period t the household is allowed to borrow new debt d_t which it will eventually repay in the next period at an interest rate r_t . The household has the option of defaulting from $t = 1$ onwards. Hence, the budget constraint of a household that repays its debt at time t is:

$$c_t + (1 + r_{t-1})d_{t-1} = y + d_t;$$

whereas, the budget constraint of a household that decides to default at time t is:

$$c_t + \gamma p_t h = y,$$

where $\gamma p_t h$ represents the renting cost, which is assumed, for simplicity, to be a fraction γ of the house's value.

¹⁸This simplifying assumption is justified by this paper's goal of understanding how different expectations about the evolution of housing prices affect agents' economic behavior and is used in several studies on the effects of housing on macroeconomic or financial decisions, as in [Campbell and Cocco \(2015\)](#) or [Cocco \(2005\)](#).

The household, then, maximizes its intertemporal utility:

$$\mathbb{E}_0 \sum_{t=0}^T \beta^t u(c_t, h),$$

subject to the period-by-period budget constraint, which is conditional on the default decision. As the amount of housing is fixed in the model, the utility function can be simplified to $u(c_t)$. Later, we will discuss in depth how agents' expectations are formed. In each period the household's choice defines a debt demand schedule $d_t(r_t)$ and a related default decision.

We can rewrite the problem recursively and solve it by backward induction. Let us then start from period $t = T$: if the household has never defaulted in the past, in the last period it is entitled to sell its housing stock; hence the only decision variable is whether to default or not to default. Since the household sells the housing stock in the last period, there is no possibility of getting new debt, and, thus, consumption is simply determined by the exogenous income and housing value.

In case of a good credit history (i.e. no past default), the problem in period T can be then written as:

$$V_T^*(r_{T-1}, d_{T-1}, p_T) = \max \{u(y - \gamma p_T h); u(y - (1 + r_{T-1})d_{T-1} + p_T h)\}.$$

Provided that the household did not default in the past, it has the option of defaulting in periods $t = 1, \dots, T - 1$. Hence, for $t = 1, \dots, T - 1$ the household has to compare two value functions: if it decides to default (or did so in the past), the value function writes:

$$V_t^D(p_t) = u(y - \gamma p_t h) + \beta \mathbb{E}_t V_{t+1}^D(p_{t+1}),$$

with $d_\tau = 0$ for $\tau \geq t$. In the event that the household did not default in the past and is not defaulting in the current period t , the value function writes instead:

$$V_t^C(r_{t-1}, d_{t-1}, p_t) = \max_{d_t} [u(y - (1 + r_{t-1})d_{t-1} + d_t) + \beta \mathbb{E}_t \{V_{t+1}^*(r_t, d_t, p_{t+1})\}].$$

Hence, in each period $t = 1, \dots, T - 1$, the household compares the two value functions to pin down its default choice:

$$V_t^*(r_{t-1}, d_{t-1}, p_t) = \max \{V_t^D(p_t); V_t^C(r_{t-1}, d_{t-1}, p_t)\}.$$

Finally, in period $t = 0$ there is no default choice, since the household is assumed to start with no debt; hence in $t = 0$ its value function reads:

$$V_0^*(p_0) = \max_{d_0} [u(y + d_0) + \beta \mathbb{E}_0 \{V_1^*(r_0, d_0, p_1)\}],$$

with the initial stock of debt $d_{-1} = 0$ given.

3.2 Bank

As the model is in partial equilibrium, the bank's behavior is modelled in accordance with the longstanding microeconomic literature on banking (see in particular [Hodgman 1960](#) and [Jaffee and Modigliani 1969](#)). The bank chooses the quantity of loans to supply to the household that maximizes its intertemporal stream of profits, taking the interest rate as given; at the same time, the bank understands that the probability of the household's default depends on the amount of debt supplied. In each period the bank obtains loans from outside the model at a risk-free rate, i_t and supplies credit to the household, at a market interest rate r_t . In case of default, the bank obtains revenues from liquidating the household's housing stock.¹⁹ The bank's problem can also be expressed in recursive form. Let's start from the last period, $t = T$. The profits for the bank write:

$$\pi_T(r_{T-1}, d_{T-1}, p_T) = \begin{cases} (1 + r_{T-1})d_{T-1} - (1 + i_{T-1})d_{T-1} & \text{if the household does not default} \\ & \text{(and did not default in the past)} \\ \kappa p_T h - (1 + i_{T-1})d_{T-1} & \text{if the household defaults} \\ & \text{(but did not in the past)} \\ 0 & \text{if the household defaulted} \\ & \text{in the past.} \end{cases}$$

Here κ represents the fraction of the collateral that the bank can recover after the household's default.

For a given interest rate r_t , in periods $t = 1, \dots, T - 1$ the bank sets d_t in such a way as to maximize its profits:

$$\max_{d_t} \pi_t(r_{t-1}, d_{t-1}, p_t) = \begin{cases} (r_{t-1} - i_{t-1})d_{t-1} + \delta \mathbb{E}_t \pi_{t+1}(r_t, d_t, p_{t+1}) & \text{if the household does not default} \\ & \text{(and did not default in the past)} \\ \kappa p_t h - (1 + i_{t-1})d_{t-1} & \text{if the household defaults} \\ & \text{(but did not in the past)} \\ 0 & \text{if the household defaulted} \\ & \text{in the past.} \end{cases}$$

By assumption, the bank cannot default on its obligations. Finally, the profit function in $t = 0$ writes:

$$\pi_0(p_0) = \delta \mathbb{E}_0 \pi_1(r_0, d_0, p_1).$$

¹⁹Notice that we model here the bank as a price-taker, but the qualitative results of the model would hold also under the assumption of the bank as a monopolist.

3.3 Recursive equilibrium

A recursive equilibrium in our economy can be defined, for $t = 0, \dots, T - 1$, as an interest rate function $r_t(p_t, d_{t-1}, r_{t-1})$, a debt function $d_t(p_t, d_{t-1}, r_{t-1})$ and value functions $V_t^D(p_t)$, $V_t^C(r_{t-1}, d_{t-1}, p_t)$ and $\pi_t(r_{t-1}, d_{t-1}, p_t)$ such that in each period $t = 0, \dots, T - 1$ and for each realization of the housing price p_t and realizations of r_{t-1} and d_{t-1} :

- given r_t , $d_t(p_t, d_{t-1}, r_{t-1})$ and value functions $V_t^D(p_t)$, $V_t^C(r_{t-1}, d_{t-1}, p_t)$ solve the household recursive maximization problem.
- given r_t and providing that no default has occurred up to period t , $d_t(p_t, d_{t-1}, r_{t-1})$ and the profit function $\pi_t(r_{t-1}, d_{t-1}, p_t)$ solve the bank maximization profit.
- markets for the consumption good and debt clear.
- in period $t = T$ the household maximizes its utility under the budget constraint, choosing whether or not to default.

It is worth to emphasize that by Walras' law, the equilibrium on the goods market determines the equilibrium on the debt market, where the interest rate r_t adjusts to its equilibrium level to equate demand and supply of credit.

3.4 Expectation Formation

In our model we treat housing prices as exogenous and assume that the growth rate of the housing price follows a stochastic process. Accordingly, given a price of housing in the initial period, p_0 , the evolution of the house price is given by:

$$p_{t+1} = p_t (1 + g_{t+1}),$$

with:

$$(1 - \Theta^p(L)) g_{t+1} = \sigma \varepsilon_{t+1}, \tag{2}$$

Here, g_{t+1} denotes the growth rate of housing price, $\Theta^p(L)$ is a lag polynomial of order $p > 1$, and ε_{t+1} is a mean-zero stochastic variable. This specification links the expectation of future house price growth rate to the autoregressive structure of the process, i.e.:

$$\mathbb{E}_t g_{t+1} = \Theta^p(L) g_{t+1}.$$

As it will be clear next section, we examine the predictions of the model when varying the form of perceived expectation on future house prices by varying the properties of the lag polynomial $\Theta^p(L)$.

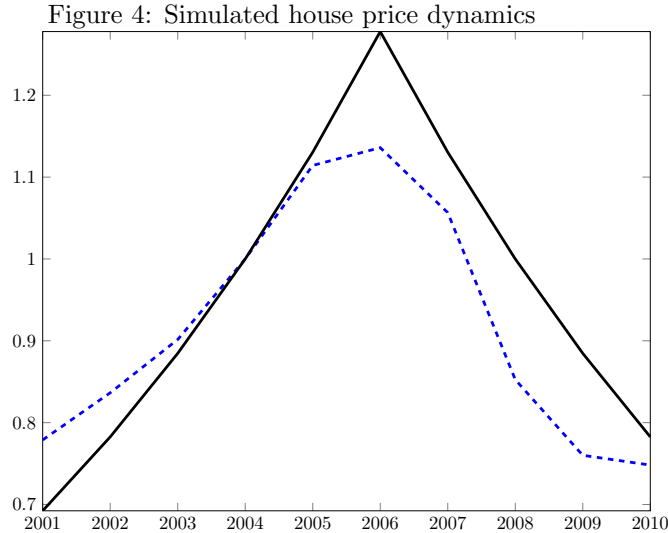
4 Calibration

By using the model described in the previous section, we now assess the quantitative effects of *natural* expectations in the consumption/saving decision. We are mainly interested in examining the extent to which the equilibrium level of housing-related debt and its price vary with the ability of agents to take into account possible long-run mean-reverting dynamics of house prices.

We consider an economy that lasts $T=10$ periods (years). The length of the simulation is a computationally restricted parameter, since in a non-stationary model the number of state-variables quickly explodes when increasing the number of periods in the model.²⁰ However, a 10-period time span is appealing for two reasons. First, it is long enough to fully capture a boom-bust episode such as the one observed in the U.S. housing market in the 2000s. Second, the majority of HELOCs started during the boom years had a duration of around 10 years.²¹

We conduct the following experiment. We feed the model with a given path of housing prices for 10 periods, which aims to replicate the boom-bust episode as experienced in the U.S. in the period 2001-2010. Then, we vary the agents' beliefs about the process generating the observed evolution of housing prices. Therefore, after observing the same initial housing price appreciation, different beliefs about the housing price data generating process affect the agents' optimal economic behavior.

The imposed evolution of housing price (solid line) is displayed in Figure 4.



Note: This figure plots the housing price series fed into the model (black solid line) along with the actual realization of the yearly average of the Real Home Price Index, available on Robert Shiller's website, from 2001 to 2010 (dotted line). The Shiller index has been rescaled and set equal to 1 in 2004.

Ultimately, we assume that agents in our model always observe the same evolution of housing

²⁰Campbell and Cocco (2015), one of the closest models to ours, is simulated over a 20-years span. However, in order to keep the state space confined, they consider a *iid* housing price growth process, approximated by a bimodal Markov process. By reducing the length of the simulation to 10 periods, we are able to consider richer housing price dynamics, allowing for an autoregressive process approximated by a tri-modal Markov process.

²¹From the Semiannual Risk Perspective From the National Risk Committee, U.S. Department of Treasury, 2012, it can be inferred that this portion was equal to at least 58 percent of loans outstanding in 2012.

prices and they rely on an autoregressive specification for the housing price growth rate in equation (2) of the form:

$$(1 - \Theta^p(L)) g_{t+1} = \sigma \varepsilon_{t+1},$$

where $\Theta^p(L)$ is a lag polynomial of order $p > 1$. To investigate the impact of different forms of expectations, we consider a large set of specifications of $\Theta^p(L)$ that generate forecasts that are similar in the short run but different in the long run. It is important to note that we are completely silent about the true process that generated the observed housing price series as this is outside the scope of our analysis. In fact, in the empirical sections above, we showed that a large set of theoretical processes are consistent with the observed historical housing price time series.

4.1 Calibrating Expectations

We consider 50 specifications for the model in equation (2) to generate agents' expectations of future housing prices. This large number of specifications allows us to investigate how macroeconomic variables respond to rather small differences in expectation formation. For computational feasibility, we limit our investigation to processes of order two, i.e.:

$$g_{t+1} = \mu(1 - \theta_1 - \theta_2) + \theta_1 g_t + \theta_2 g_{t-1} + \sigma \varepsilon_{t+1}. \quad (3)$$

Two important remarks about the choice of a second order autoregressive process are in order. First, considering a parsimonious process is paramount from computational reasons. Recall that our model is non-stationary and therefore we need to keep track of the value functions in each period. Adding more lags to the process would exponentially increase the number of state variables, making the model untractable from a computational point of view. Second, and more importantly, the AR(2) process is the most parsimonious specification that allows for hump-shaped dynamics and is flexible enough to capture features of the U.S. housing price index observed during the last boom-bust episode. Hence, by letting vary the parameters of the process in (3), we are able to match some features (mainly hump-shaped IRFs and the LRP) of a wide range of processes, such as the ones discussed in Section 2, and therefore to mimick the expectations of more or less natural agents.²²

As a result, each specification is a function of four parameters: $\mu, \theta_1, \theta_2, \sigma$. We assume that the average growth rate of housing prices, μ , is known, and it is constant across each specification. In particular, we fix $\mu = 0$, which is consistent with the historical average growth rate of the real Shiller index between 1953 and 2000, which is equal to 0.00016. We make use of three criteria to pin down the remaining three parameters ($\theta_1, \theta_2, \sigma$) for each specification. First, each specification should produce the same short-run (one-year-ahead) forecasts. This assumption is motivated by the evidence in Case et al. (2012), which find that most of the root causes of the housing bubble can be reconnected to homebuyers' long-term home price expectations. Also this assumption is motived by the fact that *natural expectations* are able to capture short-run momentum, but fail to predict more

²²For example, by setting $\theta_2 = 0$ the price pattern of a AR(1) natural agent can be recovered, whereas more negative values of θ_2 imply a lower degree of naturalness.

subtle long-run mean reversion. Second, each specification should imply the same unconditional variance. As a consequence, the different behavior implied by each specification does not depend upon the magnitude of the housing-price variance, but only upon its propagation. Third, and most important, each specification should be characterized by different long-run forecasts. As a result, each specification differs only by the degree by which it is able to capture some sort of long-run mean reversion, when keeping fixed the short-run predictions and the overall variance of the process. Specifically, we set the first order autoregressive parameter, θ_1 , to be equal to 0.6, which is the persistence of an AR(1) process estimated using the Case-Shiller index annual growth rate. Since the one-step-ahead forecasts of an AR(2) process is only a function of θ_1 , each specification implies the same one-year forecast. The long-run predictions of a model can be summarized by its long-run persistence (LRP). When considering annual data (see Table 6 in Appendix B), the LRP estimate range from the 1.5 (as estimated by the AIC model) to 2.8 (as estimated with the *intuitive* model). As Table 6 displays, there is a substantial degree of uncertainty around the estimated LRP. To capture this uncertainty, we consider specifications for process in (3) such that their LRP ranges between 1.4 and 4.5. The values of LRP in this range pin down the different values of θ_2 . Finally, the parameter σ is set to such that all specifications imply a constant standard deviation equal to the estimated value from Case-Shiller index annual growth rate (0.049). This approach allows us to isolate the effects of a change in the perceived persistence of the house price growth rate process from changes in its perceived unconditional variance. Table 3 reports the resulting calibration for six specifications of the model in equation (2) among the 50 that we consider in our simulation, together with the implied long-run persistence.

Table 3: Calibration of some processes

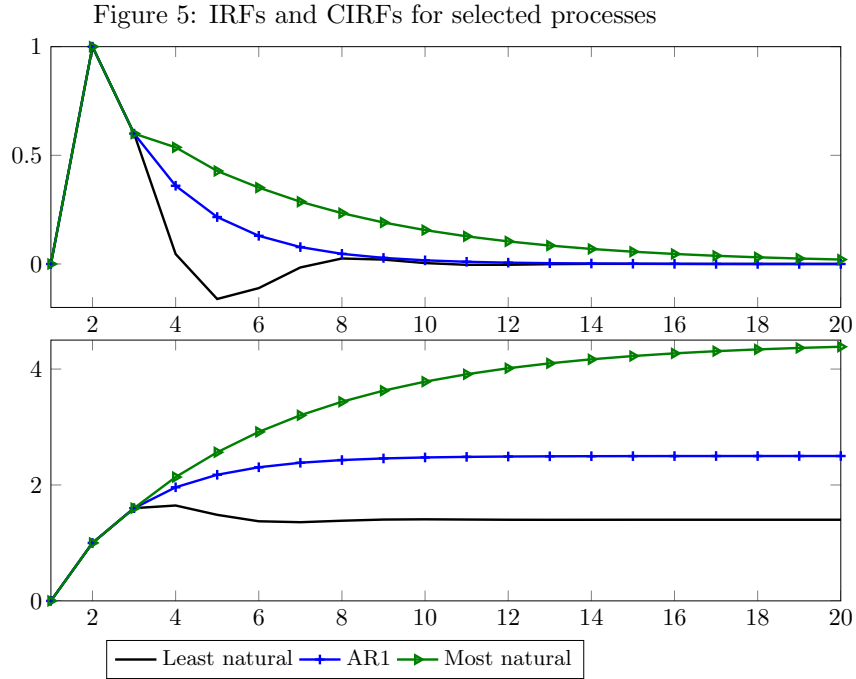
Process	LRP	θ_1	θ_2	σ
1	1.4	0.6	-0.31	0.041
10	1.93	0.6	-0.12	0.041
20	2.51	0.6	0.002	0.039
30	3.10	0.6	0.08	0.037
40	3.73	0.6	0.13	0.035
50	4.48	0.6	0.18	0.033

Note: This table reports the long-run persistence (LRP), the two autoregressive parameters (θ_1 and θ_2) and the standard deviation (σ) for six out of the 50 specifications of model as in (2).

Notice that the degree of *naturalness* of an agent is driven by the second order autoregressive parameter, θ_2 : when this parameter is negative, agents are not *natural* since they expect a long-run mean reversion of housing prices after a positive short-run momentum; when θ_2 is positive, agents are *natural* since they expect the short-run momentum to persist in the long-run.

Figure 5 displays the impulse response functions and their cumulative values for three of the above-described processes. More precisely, we plot the IRFs and CIRFs of the AR(1) process (cross-line), as a reference, along with the two “extreme” processes: process 1 (solid line) representing the process with the lowest degree of *naturalness* and which accordingly displays the strongest long-run mean reversion; process 50 (triangle-line) representing the process with highest degree

of *naturalness*. Notice that the forecasted long-run price by process 50 is almost double the one implied by an AR(1) process.



Note: This figure plots the impulse response functions for the housing price growth rate (top-panel) and level (bottom panel) for three different processes used to solve the model: the one characterizing the most natural agents (green-triangle line), the AR1 model (blue-star line), and the one characterizing the least natural agents (black-solid line).

4.2 Calibration of Structural Parameters

The calibrated structural parameters of the model and their values are reported in Table 4.

Table 4: Calibration of structural parameters

Parameter	Value	Description
β, δ	0.98	Discount rate for household and banks
h	1.5	Housing stock
η	2	CRRA coefficient
y	1	Income per year
γ	5%	Rental rate as a fraction of house value
κ	20%	Collateral value for the bank as a fraction of house value

We set the discount rate for both the household and the bank at 0.98, which implies an annual risk-free rate (i_t) of 2 percent. The housing stock, h , can be interpreted as the housing value in the initial period, since we set the initial housing price p equal to one. Hence, h relates to the housing value to income in 2000. This value is equal to 2.1 in the Survey of Consumer Finance data, whereas it is equal to 1.3 when considering national aggregate data. Hence, we set h to be equal to the intermediate value of 1.5. We assume a constant relative risk aversion (CRRA) utility function, with coefficient of risk aversion η equal to 2, a value broadly in line with the literature.

Annual income, y , is standardized at the level of 1. We assume that the rental rate, γ , is 5 percent of the current value of the housing stock, thus implying a price-to-rent ratio equal to 0.05, which is consistent with the setting in [Garner and Verbrugge \(2009\)](#) and in [Hu \(2005\)](#). Finally, we assume that when the household defaults, the bank is able to recover only 20 percent of the value of the house. Such a value is in line with our interpretation of the asset in the economy as an HELOC.²³

5 Quantitative Effects of Natural Expectations

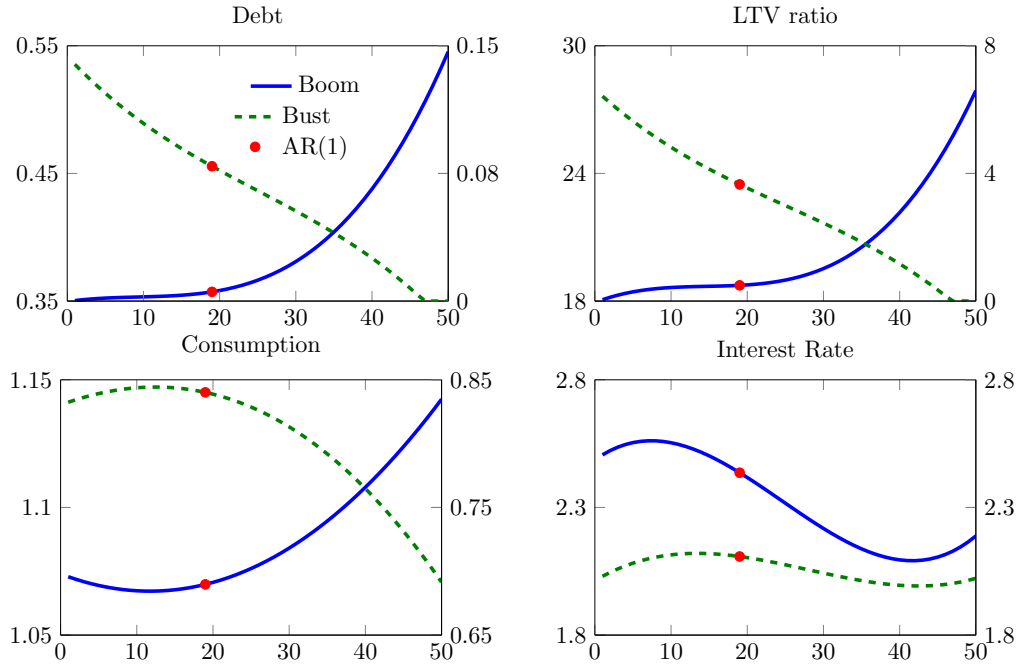
Given the calibration of the structural parameters, the 50 specifications of the housing price growth process used by agents to forecast future housing prices, and the realized evolution of housing price for the 10 periods, as shown in [Figure 4](#), we can compute the equilibrium dynamics of the variables of the model. Specifically, we are interested in the debt-to-income ratio, $\frac{d}{y}$, the consumption-to-income ratio, $\frac{c}{y}$, the loan-to-value ratio, $\frac{d}{ph}$, and the interest rate associated with home equity loans, r_t . We now investigate how these variables vary with agents' *naturalness* in the housing price boom and bust, separately.

5.1 Equilibrium in a boom

[Figure 6](#) reports the average values of debt (upper left panel), LTV ratio (upper right panel), consumption (lower left panel) and interest rate (lower right panel) for each of the 50 specifications of expected housing price growth (x-axis) across the boom phase (from period 1 to period 6 in our model, which corresponds to the period 2000-2005 in the data, blue solid line) and across the bust (from period 7 to period 9 in our model, which corresponds to the period 2007-2009 in the data, green dashed line). As a reference point, we denote with a red circle the values associated with assuming the agents form expectations using an $AR(1)$ process, which relates to the *intuitive* statistical model as presented in [Section 2](#). First, we consider the average values of our variables of interest during the boom phase.

²³Since HELOCs are junior-liens, and the maximum loan-to-value ratio for a first-lien is 80 percent, we are then implicitly assuming that the bank is able to fully recover the value of the equity in the house sale.

Figure 6: Boom and bust dynamics for selected processes



Note: This figure displays the average values of debt-to-income (upper left panel), LTV ratio (upper right panel), consumption-to-income (lower left panel) and interest rate (lower right panel) for each of the fifty specifications of expected house price growth. The values displayed in the figure have been interpolated by a 3rd degree polynomial. The x-axis reports the number of each process, from the least (process 1) to the most (process 50) *natural*. Average values are computed both across the boom phase (from period 1 to period 6 in our model, which correspond to the period 2000-2006 in the data, blue solid line) and across the bust (from period 7 to period 9 in our model, which corresponds to the period 2007-2009 in the data, green dashed line). The red nodes in each panel represents the level of debt of the AR(1) process. The y-axis for the boom phase is on the left, while the y-axis for the bust phase is on the right.

Four results are worth highlighting. First, the model predicts a positive relationship between the average equilibrium level of debt in the economy in the boom phase and the degree of *naturalness* of agents. Recall that the 50 specifications for the expectations range from higher ability of the model to incorporate long-run mean reversion (specification 1, low *naturalness*) to lower ability of the model to incorporate long-run mean reversion (specification 50, high *naturalness*). Intuitively, after observing an increase in the housing prices, a more *natural* agent expects a longer-lasting appreciation of housing prices, which gives higher incentive to demand/supply debt. In contrast, a less *natural* agent expects a short-run momentum in housing prices followed by a mean reversion adjustment after some periods, as it can be visualized by the impulse response function for specification 1 in Figure 5. As a result, the household is less willing to demand debt and the bank is less willing to supply it. A second important result relates to the role of long-run expectations. Notice when agents in the economy are characterized by the lowest degree of *naturalness*, the equilibrium level of debt is roughly 35 percent of income. In contrast, when the agents ignore hump-shaped dynamics of housing prices, the equilibrium level of debt in the economy escalates to 55 percent of income. We obtain a similar pattern when considering the loan-to-value ratio, which increases from 18 percent for the least *natural* agents to 28 percent for the most *natural* agents. The pronounced differences in these quantities is solely due to the contrasting long-run expectations of housing prices, since by construction agents have the same short-run expectations in each of the 50 specifications. These re-

sults strongly support the argument in [Case et al. \(2012\)](#): the role of homebuyers' long-run housing price expectations is a crucial determinant of agents' behavior in terms of the consumption/saving choice. As a third result, notice that the accumulation of debt fuels consumption in the short-run, since there is positive correlation among average consumption in a boom phase and the degree of *naturalness* of agents in the economy. Intuitively, when expecting higher future appreciation of house's price, the resulting wealth effect provides incentives to consume in the current period. As a fourth result, notice that debt is associated with a lower interest rate in economies where agents are more *natural*. Intuitively, since banks in the model share the same form of expectations of households, when banks expect both short-run and long-run momentum in housing prices, they are willing to lend at a lower equilibrium price.

The above findings can be summarized as follows: when housing prices start to increase, a *natural* agent (a household or a bank) overestimates the persistence of positive shocks and ignores the possible long-run mean reversion that follows a short-run momentum. As a consequence, the household or bank also overestimates the overall long-run appreciation of the housing stock. Given the availability of financial instruments to smooth future housing wealth, a *natural* household has, then, more incentive to extract a large portion of home equity to increase its consumption immediately. A *natural* bank will then be willing to provide loans to the household at lower price. As a result, *natural* expectations leads to large leverage during a housing price boom.

5.2 Equilibrium in a bust

The second set of results concerns the adjustment that the economy makes during the house price bust (periods from 6 to 9). These results reflect the predictions of our model for the behavior of agents in the period 2007-2009 and they show that the relationships between debt, consumption and degree of *naturalness* described above for the boom period are reversed. More *natural* households deleverage their debt position and they drastically reduce their consumption. Specifically, in the economies with most *natural* agents (processes 47-50), the amount of debt the household is able to extract is null.²⁴ Although quite drastic, this result is in line with evidence regarding the practice of HELOC freezes observed since 2008, when financial institutions realized the depth of the bust ([WSJ, 2008](#)). Notice that the adjustment if households were less *natural* households would be less sharp: they reduce their consumption to a lower degree and they are still allowed to borrow to smooth consumption, since they have previously accumulated relatively low levels of debt during the boom phase.

5.3 The role of bank's expectations

Since our theoretical model accounts for both the demand and supply of credit, we can now assess the impact of debt-supply *naturalness* on macroeconomic variables of interest. Specifically, we now perform some experiments to identify the contribution of banks' and households' expectations on

²⁴Such sharp dynamics in the deleveraging process may be due to the absence of frictions (e.g. adjustment costs) in lending: in case of an abrupt decline in collateral values, banks in our model suddenly cut-off lending. However, note that in the above calibration in equilibrium the household never reaches the default region.

the equilibrium outcome of debt and interest rate under the following four competing hypotheses: (a) both the bank and the household hold strongly *natural* expectations; (b) the bank and the household do not hold *natural* expectations; (c) only the household is strongly *natural*, while the bank is not; (d) only the bank is strongly *natural*, while the household is not. In these experiments, for simplicity, we give the *natural* label to an agent that forecasts future housing prices using the most *natural* process (process 50), and we give the *non-natural* label to an agent that forecasts future housing prices using the least *natural* process (process 1). These extreme values are vehicles for understanding the role of expectations in regards to supply and demand. Table 5 displays the results.

Table 5: Debt dynamics under different assumptions

	Boom		Bust	
	Debt	Rate	Debt	Rate
a) Bank and Household natural	54.5	2.2	0.0	-
b) None natural	35.0	2.5	13.9	2.0
c) Only Household natural	36.2	2.8	9.2	2.1
d) Only Bank natural	42.2	2.1	5.1	2.0

Note: This table reports the simulated average level of debt and interest rate across the boom phase (left panel, from period 1 to period 6 in our model, which correspond to the period 2000-2006) and bust phase (right panel, from period 1 to period 6 in our model, which correspond to the period 2000-2006 in the data) under the hypothesis that both the bank and household are natural (a), both bank and household are not natural (b), only the household is natural (c), and only the bank is natural (d). In this exercise, for simplicity, we assume that a natural agent uses process 50 to make forecasts, whereas a non-natural agent uses process 1.

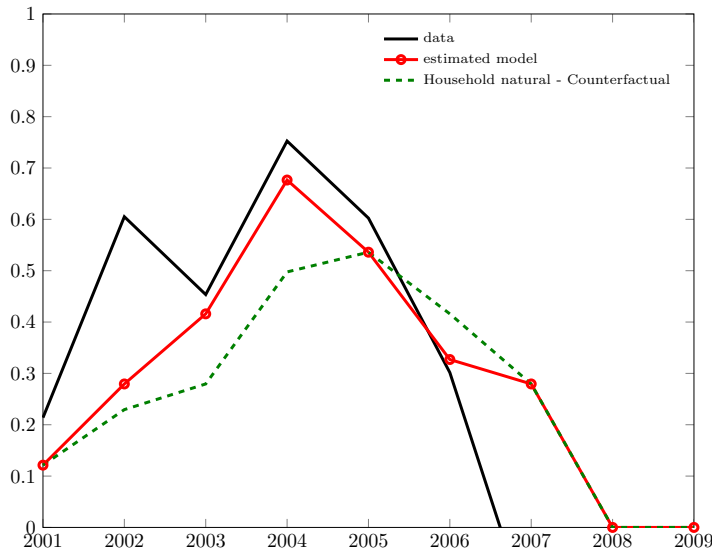
The most striking result of our experiment reflects the crucial importance of banks' expectations for the equilibrium level of debt. Let's analyze first the boom phase. When both agents are not *natural*, as in scenario (b), the equilibrium level of debt in the economy is relatively low (around 35 percent of income). If we assume that only the household is *natural*, as in scenario (c), the equilibrium level of debt increases by only 5 percent, whereas if only the bank is *natural*, as in scenario (d), the equilibrium level of debt increases up to 48 percent. In other words, without assuming a bank expectation channel, a model in which only households are natural can only replicate a small portion of the leverage level in the economy during the house price boom. The intuition for this result stems from the fact that default is more costly for the household than for the bank. When default occurs, the bank can still rely on the housing stock as a source of value: long run expectations of housing prices therefore affect intermediaries' expectations both in case of default and non-default. As a consequence, in the model, the debt supply schedule is more sensitive to long run expectations about housing prices.

5.4 Estimating Naturalness from the Data

Finally, we perform a comparison of our simulations with the debt-dynamics observed in the data to pin down which degree of *naturalness* provides a better fit. The first step is to obtain a series that is comparable to the debt-to-income ratio as simulated in our model. We first consider the annualized series of Gross Home Equity Extraction in the U.S., as in Greenspan and Kennedy

(2005).²⁵ The series is available only until 2008Q4. We divide the series by nominal disposable personal income to compute the debt-to-income ratio. Because the series is not directly comparable to the outcome of our simulated model, we need to correct the former for the fraction of households effectively extracting home equity. Therefore, we make use of the Survey of Consumer Finance data to compute the fraction of households with an outstanding HELOC and interpolate via cubic splines for the years in which the survey is not available. Such a percentage varies from 2.7 per cent in 2001 to 4.6 per cent in 2008. We then compare the resulting debt-to-income series with the debt dynamics of the model (where both household and bank can be natural) across the 50 specifications and we select the process whose debt dynamics minimize the Euclidean distance with the data. Figure 7 plots the selected process (black solid line) and the debt-to-income ratio in the data (red circled line). The selected specification is process 31, a fairly persistent and *natural* one, since its second order autoregressive parameter is positive, $\theta_2 = 0.08$, and its LRP is fairly large, equal to 3.15. Notice that the implied LRP is even higher than the one estimated on yearly data with the *intuitive* model (see Table 6 in Appendix B). To remark the importance of the bank’s naturalness, in the same figure we plot the simulated path of debt under the scenario in which only the household is natural (its expectation follow the estimated process 31) and the bank is not natural (its expectation follows the least natural process 1). It can be observed that in order to closely match the data having a *natural* household is not enough: we need a significant degree of naturalness both on the household and on the bank side.

Figure 7: Actual v. simulated data



Note: The black solid line in this figure displays the ratio of gross Home Equity Extraction over Personal Disposable Income, weighted by the fraction of households with an active HELOC (source: Survey of Consumer Finance). Data for 2007 and 2008, which take negative values, have been set to zero. The y-axis (debt to income ratio) is measured as absolute deviation from 2000 (which corresponds to our initial date $t = 0$ in the model). The red circled line is the simulated debt path arising from process 31, which is the process that minimize the Euclidian distance between the data and the dynamics of debt predicted by our model when varying the degree of naturalness of the agents (process 1 to 50). The green-dashed line represents the debt dynamics under the assumption that only the household is natural (process 31) but the bank is not natural (process 1). Sources: Greenspan and Kennedy (2005), FRED, Federal Reserve Economic Data, Federal Reserve Bank of St. Louis and SCF.

²⁵The series is the sum of (a) cash-outs resulting from refinancing, (b) originations to finance purchases of existing homes minus sellers’ debt cancellation, and (c) changes in home equity debt outstanding less unscheduled repayments on regular mortgage debt outstanding.

6 Conclusions

The Great Financial Crisis of 2007-8 has served as a reminder of the potential danger caused by undisciplined collateralized debt markets. In this paper, we used home equity extraction as a case study to explore the distortions arising from *natural* expectations about future values of collateral. We provide a simple and coherent framework of *natural* expectations for households and financial intermediaries that accurately explains the home equity extraction dynamics that arose in the boom-bust episode in the US housing market.

We developed a fairly simple model of the credit market where households and banks interact through a collateralized financial instrument and tested the model under different assumptions concerning the ability of agents to capture hump-shaped housing price dynamics. We documented that after a positive shock on housing prices, less *natural* agents expect a lower persistence of the shock. In contrast, natural agents overestimate the persistence of the process, thus leading to overly optimistic long-run forecasts. We then simulated the model by considering housing price dynamics as observed during the 2000s. Our model predicts a positive relationship between the amount of home equity extracted in a boom phase and the degree of *naturalness* of the agents in the credit market, while at the same time stressing the prominence of banks' expectations in the equilibrium outcome. We highlight that financial intermediaries' *naturalness* is a crucial component for observing a large accumulation of debt at low interest rates and that the dynamics of U.S. home equity extraction during the recent boom and bust can be replicated only when both households and financial intermediaries agents hold *natural* expectations.

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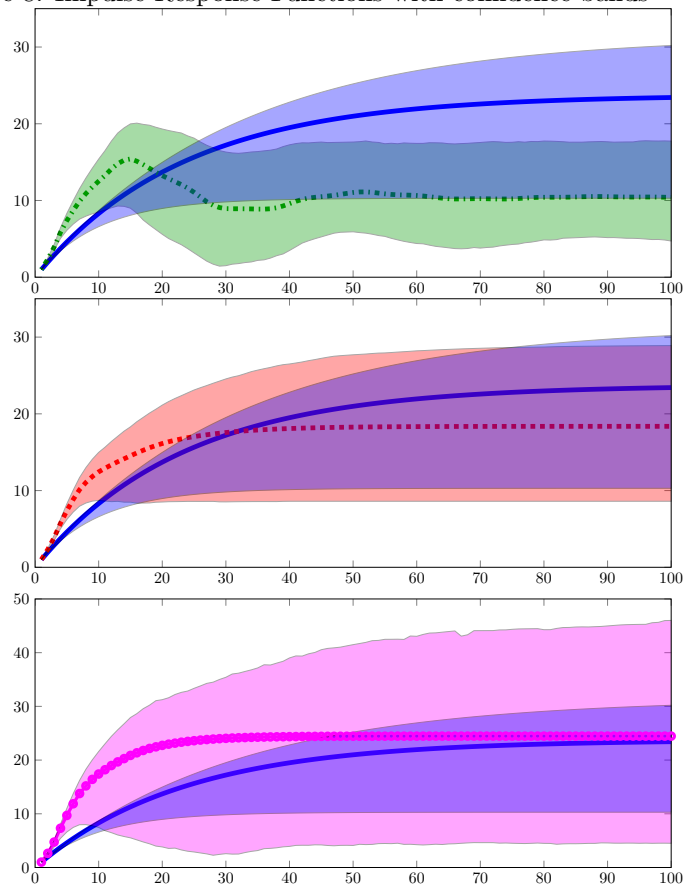
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A Appendix: Confidence Band Impulse Response House Price

The top panel of Figure 8 plots together the level impulse response of the *intuitive* model (blue solid line) and the AIC model (green dotted line) and their 95 percent confidence band (shaded area); the central panel plots together the level impulse response of the *intuitive* model (blue solid line) and the BIC model (red dashed line) and their 95 percent confidence band; and the bottom panel plots together the level impulse response of the *intuitive* model (blue solid line) and the *finite memory* model (purple circled line) and their 95 percent confidence band. As expected, the uncertainty around the impulse responses is large and the confidence bands largely overlap.

Figure 8: Impulse Response Functions with confidence bands



Note: This figure reports the cumulative impulse response function (CIRF) of house price growth rate to a positive unitary shock. Shaded areas represent the 95 per cent confidence intervals. Top panel: intuitive model (blue solid line) and AIC model (green dotted line). Central panel: intuitive model (blue solid line) and BIC model (red dashed line). Bottom panel: intuitive model (blue solid line) and finite memory model (purple circled line).

B Appendix: Long-Run Price for Annual Data

Table 6: LRP and Confidence Band

p	Natural	BIC	AIC	Short Memory
	1	6	7	2
Long-Run Persistence (LRP)	2.76	1.72	1.52	2.29
Confidence Bands (95%)	[2.17;4.49]	[0.85;3.05]	[0.67; 2.91]	[0.25; 5.17]