Endogenous Partial Insurance and Inequality*

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February 23, 2018

Abstract

In this paper, we propose a model of endogenous partial insurance and we investigate how it influences macroeconomic outcomes, such as wealth inequality, social mobility and consumption smoothing. To this purpose, we introduce participation costs to state-contingent asset markets into an otherwise standard Aiyagari (1994) model and we show that endogenous partial-insurance may lead to a large increase in wealth inequality, predicts a heterogenous degree of insurance consistent with the empirical findings in Guvenen (2007) and Gervais and Klein (2010), and generates an overall level of insurance in line with the estimate in Guvenen and Smith (2014). The key insight behind these results stems from the non-monotonic relationship between wealth and desired degree of insurance, when insurance is costly. Poor borrowing constrained households remain uninsured, middle-class households are perfectly insured, while rich households decide to self-insure by purchasing risk-free assets.

JEL codes: E21, E44.

Keywords: Wealth Inequality, Participation costs, Endogenous Partial Insurance.

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1 Introduction

Recent papers have underscored important stylized facts about the heterogenous degree of risk-sharing and consumption smoothing across US households: using PSID data Guvenen (2007) documents that stockholders smooth less consumption than non-stockholders; similarly, using CEX data Gervais and Klein (2010) find that households with larger financial assets smooth consumption less than households with lower financial assets.[1] These facts pose a problem for standard heterogenous agents models, since, as already noted by Broer (2013), the self-insurance model, as in Aiyagari (1994), and the limited commitment model, as in Krueger and Perri (2006), are not able to capture the observed heterogenous degree of insurance.

This caveat couples with other well-known issues of the conventional Aiyagari incomplete market model. First, it fails to deliver a strong amplification from income to wealth inequality when it is characterized only by reasonably calibrated income shocks, as summarized in Quadrini and Rios-Rull (2014).[2] Second, it implies an aggregate level of consumption insurance that is much lower than what is estimated in the data, as pointed out in Guvenen and Smith (2014).

In this paper we first propose a very tractable model that generates endogenous partial insurance from a generalization of the standard Aiyagari model and, then, we show that the existence of endogenous partial insurance is, *per-se*, able to: (i) generate a large level of wealth inequality from income shocks that would otherwise imply very little inequality in the standard Aiyagari model; (ii) generate a heterogenous degree of consumption smoothing across the wealth distribution in line with the empirical findings of Guvenen (2007) and Gervais and Klein (2010); and (iii) generate an aggregate level of insurance that is larger than in Aiyagari and that is closer to the value estimated in Guvenen and Smith (2014).

Our first contribution is to propose a simple model of endogenous partial insurance. In our setting, markets that potentially provide full insurance do exist, but it is costly to access to them. More precisely, in an otherwise standard general equilibrium economy as in Aiyagari (1994), we introduce costs for participating in contingent asset markets.[3]

[1]This result is robust to restricting the sample only to working age heads of the household persons, to excluding households living in rural areas, and to excluding self-employed households.

[2]Many authors have extended these models to improve the ability to generate greater wealth inequality. Among these approaches are the addition of special earning risks (Castaneda et al. (2003), Benhabib et al. (2015)), entrepreneurial risks (Quadrini, 2000; Cagetti and De Nardi, 2009; Angeletos, 2007; Buera, 2009)), bequest, human capital, and health risk (De Nardi (2004), Huggett (1996)), stochastic discounting (Krusell and Smith (1998)), and capital income risk (Benhabib et al. (2011)).

[3]The idea that consumption smoothing is costly underpins our approach: being active in financial markets involves monetary costs, broadly defined, such as fees and transactions costs charged by brokers

Consequently, households face a trade-off between paying the participation cost and enjoying the gain of consumption smoothing. Conveniently, our model nests, as polar cases, both the complete market model, henceforth labelled as *perfect-insurance equilibrium*, in which the participation cost is so low that all agents optimally decide to provide insurance to each other, and the standard incomplete market model as in Aiyagari (1994), henceforth labelled as *self-insurance equilibrium*, in which the cost is so high that all agents prefer to only accumulate risk-less assets as consumption buffer.[4] However, more generally, intermediate levels of participation costs lead the economy to a *partial-insurance equilibrium*, in which only a fraction of the population endogenously decide to fully insure. We show that under very general condition on the utility function the degree of insurance is non-monotone across wealth: poor people are the least insured, the middle class the most insured, slightly more than the richest. Hence, our endogenous partial insurance mechanism rationalizes the findings of Guvenen (2007) and Gervais and Klein (2010).

To provide intuition on the endogenous insurance decision, we first investigate a simple insurance model similar to the one in Kimball (1990b). We highlight that when the utility function features prudence (negative third derivative) agents' insurance motives may lead to pay the cost, but when it features also decreasing absolute prudence (positive forth derivative) a positive participation cost deters the richest to trade contingents assets. Importantly, our analysis demonstrates that the heterogeneity of insurance with respect to wealth is a quite general result since, as discussed in Kimball (1990a), commonly used parameterizations of the utility function, such as the constant relative risk aversion utility, display decreasing absolute prudence.

We then incorporate the endogenous insurance decision into a standard neoclassical model with idiosyncratic shocks as in Aiyagari (1994). We assume that two types of assets are available in the economy: a set of state contingent assets, which can be purchased only by paying a fixed participation cost, and a risk-free asset. Hence, agents first decide whether they want to participate in the financial markets, and, then, they decide against which states they are willing to buy insurance. We first demonstrate that when varying the participation cost, the model is characterized by a continuum of partial-insurance equilibria, in which the *perfect-insurance equilibrium* and the *self-insurance equilibrium* are the polar cases. We prove that households decide to participate in a contingent market as long as its participation cost is lower than a certain threshold value, which depends positively on the households' gains of insurance, and, when the utility function features

and intermediaries, costs related to information acquisition, and non-monetary costs, such as the opportunity cost of time devoted to find the best portfolio allocation. See Section 6 for further discussion. See also Acemoglu and Zilibotti (1997) for the role of fixed cost on capital accumulation and growth.

[4]This is obviously equivalent to assuming that state-contingent assets do not exist, as in Aiyagari (1994).

decreasing absolute prudence, it depends non-monotonically on households' wealth. As a result, the *partial-insurance equilibrium* is characterized by a set of poor households that are not able to obtain any insurance, by a set of middle-class households that actively participate in the contingent asset market and, hence, are fully insured, and, interestingly, by a set of rich households that prefer to self insure by accumulating a large stock of the risk-free assets.

Our second contribution is to quantify the effects that endogenous *partial-insurance equilibrium* has on aggregate implications for inequality, social mobility, asset prices, and degree of insurance. The first of these implications concerns inequality. When participation costs reduce from a arbitrary large value, such that the economy is equivalent to a *self-insurance equilibrium*, to intermediate values, such that the economy turns into a *partial-insurance equilibrium*, wealth inequality can dramatically increase.[5] With intermediate values of participation costs our model can predict a level of wealth inequality similar to the one observed in the U.S. data (Gini index equal to 0.93). Notice that our calibration employs the same income shock structure that would otherwise imply very small wealth inequality (Gini index equal to 0.12) in the standard Aiyagari model.[6] As a result, endogenous partial insurance allows to obtain large wealth inequality in a model with just reasonably calibrated income shocks. There are two effects that rationalize this result. First, perfectly insured middle-class households do not have incentive to accumulate more assets for insurance purposes, while the richest ones do. This feature skews upward social mobility so that middle-class agents are less likely than richest agents to increase their wealth and, as a result, the upper tail of the wealth distribution thickens in presence of intermediate levels of participation costs. Second, a general equilibrium effect reinforce the skewness of the wealth distribution since in a model with partial-insurance the interest rate is larger than in Aiyagari (1994)'s model. In our quantitative exercise we isolate these effects. In addition, we show that our model predicts that a very large of share of wealth is concentrated in the hands of the top percentiles of the wealth distribution; this feature is in line with the data, and cannot be generated by a standard Aiyagari (1994)'s model that features the same calibration of the income process.

Furthermore, we can then draw a similarity between our *partial-insurance equilibrium* and the degree of partial insurance discussed in Guvenen and Smith (2014). In fact, in our model participation costs lead some households to choose to be perfectly insured and

[5]This result, then, links the increased innovation in the financial sector in the last three decades, as documented by Lerner (2002) to the increased wealth inequality in the same time span, as reported by Saez and Zucman (2014).

[6]Hence, our model can predict a large wealth inequality starting with a much less disperse income process than in Castaneda et al. (2003). For example, income dispersion, measured as Gini index on income, in our model is 0.097, whereas it is 0.600 in Castaneda et al. (2003).

some other households to choose to be only self-insured. Hence, the fraction of population that is perfectly insured is a function of the level of participation costs. Hence, whereas in Guvenen and Smith (2014)'s setting partial insurance is on the intensive margin - agents can insure a fraction of their income, in our setting partial insurance is on the extensive margin - agents can be insured or not. Using different calibrations of the model, we show that degrees of partial insurance above and below the one estimated by Guvenen and Smith (2014), around 45 percent, can lead to the realistically observed wealth inequality in presence of participation costs.

To obtain these results, we focus on the simplest structure of cost, where households pay a unique fixed cost to access all contingent asset markets. More specifically, households have to pay $\sum_i q_i a_i + \kappa$ to purchase a_i bonds contingent on future state i, where q_i denotes the price of the asset, and κ denotes the additional fixed participation cost. Yet, there is no loss of generality to focus on this specific structure. In particular, we show how to extend our results to asset-specific cost in Appendix C. In particular, this alternative structure implies a new decision so as to decide the order of states against which the household decides to get insurance. Yet, this richer structure of partial insurance does not yield different macroeconomic outcomes. More fundamentally, our results only rely on a smaller degree of partial insurance of richest agents compared with the middle-class, which we connect to the lower desire of insurance with respect to wealth in the presence of costly insurance.

Related literature. In addition to the papers that we have already mentioned, our work expands on several bodies of the literature.

Among the empirical studies conducted on lack of insurance and consumption smoothing as Townsend (1994) and Mace (1991), our work bears similarity to that of Cochrane (1991), and, more recently, Grande and Ventura (2002), who study households' insurance against different types of risk. They show that households are well insured against certain types of risks, such as health problems, but not against other types of risks, such as unemployment (especially involuntary job loss) (see also Blundell et al., 2008).

Our work also amplifies on the literature linking models of incomplete insurance with empirical evidence as in Krueger and Perri (2005, 2006) or Kaplan and Violante (2010), who assess the degree of insurance beyond self-insurance. In our setting the participation cost modifies the link between income and consumption inequality, through the resulting non-monotone degree of insurance across wealth. Hence, trends in one of these variables are imperfectly transmitted to the other, consistently with the findings in Attanasio et al. (2012) and Aguiar and Bils (2015).

Finally, our work links to the literature in finance on limited participation as in

Luttmer (1999), Vissing-Jorgensen (2002) and more recently in Paiella (2007), Guvenen (2009) or Attanasio and Paiella (2011) among others. In these models, the access to the stock market is costly or open only in a subset of periods. Also, even when economists focus on limited asset trading,[7] they generally do not consider frictions related to asset market participation in their models.

The rest of the paper is organized as follows: in Section 2 we present a simple insurance model in order to provide conditions and intuitions for households' insurance decision. In Section 3 we describe the general economic environment. Section 4 describes how our model of endogenous partial insurance differs from standard Aiyagari models Section 5 presents the quantitative results about wealth inequality. Section 6 discusses a set of further extensions. Finally, Section 7 provides concluding remarks.

2 A Simple Insurance Model

In order to gain some intuition about households' individual contingent-market participation choice, we first analyze a simple two-period and two-state insurance model. Our model is similar to the one proposed in Kimball (1990a) and in Kimball (1990b), in which we include a fix cost to state contingent asset market participation. We will show that, in presence of participation costs for trading contingent assets, weak conditions on consumers' utility function lead to rich agents to be better off by not participating in insurance markets.

The economy lasts two periods, $t = 0, 1$. The household is endowed with a level of wealth W in both periods. In period $t = 1$ the household might face an exogenous loss of wealth, $-L \geq 0$, which occurs with probability p. With probability $1 - p$, the household receives a positive shock $pL/(1 - p)$ so that the expected loss is 0. The indicator variable 1_L describes the realization of the state of nature. Let define as *feasible* the levels of wealth such that $W > L$, to assure that consumption is positive in every period and in every state. The household maximizes the following expected utility function: $E_0 (u(c_0) + u(c_1))$. For simplicity, we assume that there is no discounting.

We introduce an endogenous decision of participating in the insurance market. The agent has access to state contingent assets. At time $t = 0$, the agent can acquire α units of a state-contingent asset at unit price q_α that repays a unit of consumption good at time $t = 1$ only if the loss in wealth occurs, i.e. if $1_L = 1$, and β units of a state-contingent asset at unit price q_β that repays a unit of consumption good at time $t = 1$

[7] This can happen because of lack of commitment (Thomas and Worrall, 1988; Kocherlakota, 1996), trading technologies (Chien et al., 2011) or because of *ad hoc* assumptions as in the incomplete market literature.

only if the loss in wealth does not occur, i.e. if $1_L = 0$. Importantly, in order to have access to the state contingent asset, the agent needs to pay a fixed cost κ. The household is not necessarily willing to pay the fixed cost and, hence, we define $\delta(W, \kappa)$ as a choice variable that denotes the contingent asset market participation, given a level of wealth and a participation cost: if the household pays the cost and purchases contingent assets, $\delta(W, \kappa)$ equals 1. Otherwise, it equals 0.

Remark. In this simple model the alternative of perfect insurance is autarky. We will illustrate the condition such that richer agents prefer not to be fully insured. That condition holds also when the alternative of perfect insurance is accumulating only a risk free asset for self-insuring, as standard in an Aiyagari (1994) model and as studied in the next session.

Conditional on participation, $\delta(W, \kappa) = 1$, the budget constraints are:

$$c_0 + q_\alpha \alpha + q_\beta \beta + \kappa = W$$
$$c_1 = W + 1_L(\alpha - L) + (1 - 1_L)(\beta + pL/(1 - p)).$$

Without loss of generality, we assume that the prices of contingent assets are actuarially fair, ($q_\alpha = p$ and $q_\beta = 1-p$). In this case, the optimal amount of insurance is: $\alpha = L-\kappa/2$ and $\beta = -\kappa/2 - pL/(1 - p)$, and the agent's expected utility is:

$$V^P(W, \kappa) = 2u\left(W - \kappa/2\right),$$

where the superscript P denotes the expected utility of an agent that participates in the insurance market.

Conditional on no-participation, $\delta(W, \kappa) = 0$, then $\alpha = 0$, $\beta = 0$, and the two periods budget constraints are:

$$c_0 = W$$
$$c_1 = W - 1_L L + (1 - 1_L)pL/(1 - p).$$

The expected utility of the agent is:

$$V^N(W) = u(W) + \left[(1 - p)u\left(W + \frac{pL}{1 - p}\right) + pu\left(W - L\right)\right],$$

where the superscript N indicates the utility of an agent that does not participate in the insurance market. Let $\mathbb{P}(\kappa)$ be the set of wealth levels for which participation in the insurance market is optimal for a given participation cost κ. Formally:

Definition 1. (Participation Set). For a given participation cost κ, for any wealth level in $\mathbb{P}(\kappa)$ insurance market participation is optimal, that is:

$$\mathbb{P}(\kappa) = \left\{W \in (L, \infty) : V^P(W, \kappa) > V^N(W)\right\}.$$

Let define the gain of insurance as $G(W,\kappa) = \frac{1}{2}\left(V^P(W,\kappa) - V^N(W)\right)$. It can be rewritten as:

$$G(W,\kappa) = u\left(W - \frac{\kappa}{2}\right) - \frac{1}{2}u(W) - \frac{1-p}{2}u\left(W + \frac{pL}{1-p}\right) - \frac{p}{2}u\left(W - L\right). \qquad (1)$$

The first set of results concern the frictionless economy with no costs.

Proposition 1. *(Insurance Incentives without cost) Let $u(x)$ be a three-times continuous and differentiable utility function, such that $u'(x) > 0$, $u''(x) < 0$, and satisfies the Inada conditions: $\lim_{x\to\infty} u'(x) = 0$, and $\lim_{x\to 0} u'(x) = \infty$. Then, for any feasible level of wealth, i.e. $\forall W > L$:*

1. *$G(W,0) > 0$;*
2. *$\lim_{W\to\infty} G(W,0) = 0$.*
3. *If $u''' > 0$ then $\frac{\partial G(W,0)}{\partial W} < 0$.*

Proof. See Appendix D.1 for the proof. □

Proposition 1 shows that, absent any cost, $\kappa = 0$, the (strictly) concavity of the utility function guarantees a (strictly) positive benefit from insurance. If the utility function has a positive third derivative, its marginal utility is convex and, therefore, displays *prudence*, as defined in Kimball (1990b), and a decreasing absolute risk aversion. In this case, the gains from insurance $G(W,0)$ are decreasing with respect to wealth. As discussed in Kimball (1990a), *prudence* measures the strength of the precautionary saving motive, which induces individuals to prepare and forearm themselves against uncertainty they cannot avoid- in contrast to risk aversion, which is how much agents dislike uncertainty and want to avoid it.

We now consider the economy with participation costs.

Proposition 2. *(Insurance Incentive with cost) Let $u(x)$ be a four-times continuous and differentiable utility function, such that $u'(x) > 0$, $u''(x) < 0$, $u'''(x) > 0$, $u''''(x) < 0$, and satisfies the Inada conditions: $\lim_{x\to\infty} u'(x) = 0$, and $\lim_{x\to 0} u'(x) = \infty$. Then, for any feasible level of wealth, i.e. $\forall W > L$, and for any feasible level of cost, i.e. $\kappa < 2L$:*

1. *(Existence of Thresholds). Let $\hat{\kappa} < 2L$ be the solution of $G(L,\hat{\kappa}) = 0$. Then, $\forall \kappa < \hat{\kappa}$, $\exists!\ \overline{W}(\kappa) > L$: $W \in \mathbb{P}(\kappa) \iff L < W < \overline{W}(\kappa)$.*
2. *(Comparative static of participation set)*
   - *Participation set coincides with all feasible wealth levels when $\kappa = 0$, that is:*

$$\mathbb{P}(0) = \{W : W > L\}.$$

- *Participation set is shrinking in participation cost, that is for all $\kappa^1 < \kappa^2$, if $W \in \mathbb{P}(\kappa_2)$ then $W \in \mathbb{P}(\kappa_1)$; hence, $\mathbb{P}(\kappa_2) \subset \mathbb{P}(\kappa_1)$.*
- *Participation set is empty for any participation cost greater than $\hat{\kappa}$, that is: $\forall \kappa > \hat{\kappa}$, $\mathbb{P}(\kappa) = \emptyset$.*

Proof. See Appendix D.2 for the proof. □

This proposition explains a crucial characteristic of the insurance market. When accessing to the insurance market is costly, the agent endogenously decides whether to participate in that state-contingent asset market depending on the level of its wealth. When wealth level is large enough, $W > \overline{W}(\kappa)$, the agent is better off by not-participating in the insurance market since the cost of paying the fix cost is larger than the expected benefit of reducing the loss in case of occurrence of the negative shock. For those wealth levels, in fact, $G(W, \kappa) < 0$. Also, notice that the participation set varies with the participation cost. When the cost tends to zero, the participation set corresponds to the entire feasible wealth domain. On the contrary, the participation region disappears when the cost is larger than a certain threshold $\hat{\kappa}$. In this case entering in the insurance market is either infeasible or not beneficial. The necessary condition for the existence of the threshold wealth level is a strictly negative forth derivative of the instantaneous utility function. This condition is equivalent to assume a utility characterized by *decreasing absolute prudence*. As described by Kimball (1990a), which relates this assumption to precautionary saving behavior, approximate constancy for the wealth elasticity of risk-taking is enough to guarantee decreasing absolute prudence. Also, commonly used parameterizations of the utility function, such as the constant relative risk aversion utility, displays decreasing absolute prudence.

Remark. Notice that holding a combination of the two assets is equivalent to holding a risk-free asset. Generally, the results in this section will be the same if, instead, we assume the existence of a risk-free asset and only one contingent-asset that is subject to participation cost. In that case, the risk-free asset can be accumulated to do precautionary savings, but it does not provide full insurance. Hence, in that case the participation decision driven by enjoying the gains of full insurance is similar to our setting, which we chose to be as close as possible to Kimball (1990a).

3 A Model of Endogenous Partial Insurance

In this section we describe the general economic environment. We consider an infinite horizon production economy populated by a continuum of mass 1 of *ex ante* homogenous households. This model follows closely Aiyagari (1994) except for two dimensions: we introduce securities contingent to idiosyncratic states and we simultaneously introduce fixed participation costs for each contingent market. Time is discrete and indexed by $t \in \{0, 1, ...\}$.

Uncertainty and preferences. Each Household chooses consumption so as to maximize the following utility: $U = E \sum_{y^t} \beta^t \pi(y^t) u\left(c(y^t)\right)$, where $\beta \in (0, 1)$ is the discount factor, $c(y^t)$ denotes consumption at date t, and u is a strictly increasing and concave function that satisfies $\lim_{c \to 0} u'(c) = -\infty$ and $\lim_{c \to \infty} u'(c) = 0$. Without loss of generality, u is twice differentiable.

Households inelastically provide labor. At every period they receive a stochastic labor endowment, y_t. Since there is no aggregate uncertainty, this assumption is equivalent to consider that households receive a stochastic good-endowment $\tilde{y}_t = wy_t$, where w is the constant wage rate.

We assume that y_t follows a Markov process, which takes values in $Y = \{y_1, ...y_N\}$ and that $\pi(y_j|y_k)$ is the associated transition probability from state j to state k. We denote by y^t the history of the realizations of the shock, $y^t = \{y_0, y_1, ..., y_t\}$, and by $\Pi(y_k)$ the fraction of households in state k.

Asset structure. To smooth consumption, households may trade a set of different assets. First, they can purchase non-contingent bonds. Each of these bonds yields, unconditionally, one unit of goods next period. We denote by $B(y^t)$ household's position in the risk-free assets and by q^f its price. Besides, as in Aiyagari (1994), we impose that this position is bounded below: $B(y^t) \geq -\overline{B}$ where $\overline{B} \geq 0$ is finite.[8] Second, households can trade a set of $N - 1$ state-contingent assets.[9] In state y_m, each of these assets pays contingently to the realization of y_k next period: it pays 1 when $y = y_k$ and 0 otherwise. We denote by $q(k, m)$ the price of this asset and by $a(k, y^t)$ the corresponding holdings of a household with history of shocks y^t. Note that in our notation contingent asset holding depends on the current state m through the history of shock y^t. As for the risk-free asset, we assume the existence of ad-hoc constraints for the state contingents assets, i.e. $a(k, y^t) \geq \bar{a}$, to rule out unbounded positions.[10]

The novelty we introduce in this paper is that purchasing those assets requires paying

[8] We do not provide further foundations for that constraint. It can be exogenous debt limits as in Bewley (1980), natural debt limits as in Aiyagari (1994) or endogenous borrowing constraints as in Zhang (1997) or Abraham and Carceles-Poveda (2010) for such foundations.

[9] Since there are N possible state, one risk free asset and $N - 1$ state contingent assets, the asset structure may be able to span all possible consumption plans.

[10] Alternatively, the borrowing constraint can be written on the joint stock of non-contingent and contingent assets and this may allow households to gamble with contingent assets.

a fixed fee, κ. Hence, in order to hold $a(k, y^t)$ units of any contingent assets household has to pay $q(k, m)a(k, y^t) + \kappa$. Here, for simplicity, we assume that if the agent pays the participation cost she can purchase or sell the preferred quantity of any state contingent assets. We assume that κ is a pure waste.[11]

The presence of the fixed cost implies that the household needs to take a discrete decision about whether to participate in the contingent asset market. We denote by $\delta(y^t) \in \{0, 1\}$ the corresponding decision variable, with the following meaning: when $\delta(y^t) = 1$, household with history y^t decides to enter in the state-contingent assets and when $\delta(y^t) = 0$, she does not.

In the end, the proceeds of both contingent and risk-less assets are invested in physical capital, whose returns are used to honor assets' payments.

Remark. The borrowing constraint introduces a limit to markets, even when participation costs are absent. Markets are then not complete *stricto sensu*. Yet, we will show that there are complete *de facto*, in the sense that the borrowing limit does not prevent full households' insurance.

Remark. The main results of this paper hold when assuming that participation cost is state-dependent, κ_j. In this case, households' decides in which state-contingent asset market to enter and, therefore, the participation decision is a set of binary variables. In Appendix C, we present this setting.

In the end, a household with a history of shock y^t and a current shock realization y_m faces the following sequence of budget constraints:

$$c(y^t) + q^f B(y^t) + \delta(y^t) \left(\sum_k q(k, m)a(k, y^t) + \kappa \right) = B(y^{t-1}) + a(m, y^{t-1}) + wy_m.$$

Recall that in case of non-participation, $\delta(y^t) = 0$, the household is excluded from the contingent-asset market, and, therefore, in that case $a(k, y^t) = 0$.

Production. As in Aiyagari (1994), we include production in our economy, creating an endogenous net supply of assets. A single representative firm produces using a Cobb-Douglas technology:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t,$$

[11]This involves no loss of generality. In a more general setting, where transaction costs may be pecuniary costs charged by intermediaries, fixed costs paid by some agents will be other agents' revenues. Here, our assumption is close to assuming a redistribution of intermediaries' profits to households in a lump-sum way.

where capital, K_t, and total labor, L_t, are rent from households. Total labor is the combination of labor provided by the different types of households ($y = y_k$, for $k = 1, .., N$), i.e.:

$$L_t = \sum_k \Pi(y_k) y_k.$$

First order conditions for capital and labor are:

$$A\alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} = r + \delta \text{ and } (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} = w.$$

Market clearing condition. The asset market-clearing condition pins down aggregate capital, K_{t+1}, as:

$$K_{t+1} = \sum_{y^t} \sum_k \left(q(k,m) a(k, y^t) + q^f B(y^t) \right),$$

and the goods market-clearing condition pins down aggregate consumption, C_t, as:

$$C_t + \sum_{y^t} \delta(y^t)\kappa = \sum_{y^t} \left[c(y^t) + \delta(y^t)\kappa \right] = Y_t - K_{t+1} + (1-\delta) K_t.$$

Recall that in our notation the current individual state m is included in the history of shocks y^t. As agents are *ex ante* homogenous, the sum over all histories y^t amounts to sum over all the individuals.

Recursive formulation. In this setting, the problem faced by households is complex: it integrates a double maximization to decide about participation in the contingent asset market and about asset purchases. Formally, this problem can be written as follows:

Problem 1.

$$\max_{\delta(y^t), c(y^t), B(y^t), a(y^t)} \sum_{y^t} \beta^t \pi(y^t) u(c(y^t))$$

$$\text{s.t. } c(y^t) + q^f B(y^t) + \delta(y^t) \left(\sum_k q(k,m) a(k, y^t) + \kappa \right) = w y_m + B(y^{t-1}) + a(m, y^{t-1}),$$

$$B(y^t) \geq -\overline{B}, \qquad a(k, y^t) \geq 0, \qquad \text{and } a(y^t) = 0 \text{ if } \delta(y^t) = 0.$$

Fortunately, this problem can be rewritten recursively. Indeed, in Appendix B we show that it is equivalent to solve the following problem, for which the value function V is unique:[12]

[12]This means that the discrete choice does not prevent the existence and uniqueness of the value function.

Problem 2 (Recursive formulation)**.** Given $\{w, q, q^f\}$,

$$V(x, B, \{a(y)\}, y) = \max_{\delta \in \{0,1\}} \max_{\{a'\}, B'} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(x', B', a'(y'), y') \right\}$$

$$\text{s.t. } c + \delta \left(\sum_{y'} q(x, y', y) a'(y') + \kappa \right) + q^f(x) B' \leq w(x) y + B + a(y),$$

$$B' \geq -\overline{B}, \qquad a'(y') \geq 0, \qquad a'(y') = 0 \text{ if } \delta = 0, \text{ and } \qquad x' = H(x),$$

with solution $\{\delta, \{a'(y')\}, B'\} = h(x, B, \{a(y)\}, y)$.

With this notation we emphasize that agents decide on a portfolio of Arrow securities, denoted with $\{a'\}$. However, only the Arrow security associated with the realized state matters, and, therefore, the state variable for the current value function is $a(y)$.

Agents are indexed by $\{B, \{a\}, y\}$, describing their asset positions as well as their labor supply. We denote by x the probability measure over Borel sets of compact set $S = Y \times A$, where A is the set of households' asset positions. As in Davila et al. (2012), we can construct the aggregate law of motion. To this purpose, we first construct the individual transition process. Let $J \in S$ be a Borel set. The corresponding individual transition function is:

$$Q(x, B, \{a\}, y, J, h) = \sum_{y' \in J_{y'}} \pi(y'|y) \xi_{h(x,B,\{a\},y) \in J_{\{B,\{a\}\}}},$$

where ξ is the indicator function. As a result, we can define the updating operator $T(x, Q)$ for tomorrow's distribution, x', given today one, x:

$$x'(J) = T(x, Q)(J) = \int_S Q(x, B, \{a\}, y, J, h) dx.$$

Finally, we can define the equilibrium in a recursive way:

Definition 2. A *recursive competitive equilibrium* is a pair of function h and H that solves problem 2 given H and such that $H(x) = T(x, Q(.; h))$.

4 Comparison with the Aiyagari model

This section characterizes the equilibrium outcome of our model and emphasizes how this differs from a standard Aiyagari (1994) model. First, we focus on partial equilibrium; by extending the simple endogenous insurance decision model proposed in Section 2 to an infinite-horizon setting, we show that the existence of participation cost for contingent assets implies a non-monotone consumption smoothing, and equivalently a non-monotone degree of insurance, as function of wealth. This feature contrasts the implications of the Aiyagari (1994)'s model, in which consumption smoothing improves with wealth under standard preferences. Then, we move to general equilibrium analysis; we show that when varying the participation cost κ, the model is characterized by a continuum of equilibria ranging from the one equivalent to the complete market model (when κ is low enough) to the standard incomplete markets à la Aiyagari (1994) (when κ is high enough). We further show that the equilibrium real interest rate varies continuously from the time discount rate, β, to the value in the Aiyagari (1994) economy, the resulting liquidity premium of capital, thus capturing the aggregate degree of partial insurance.

To make the exposition simpler, in this section we assume that the productivity shock follows a two-state first-order Markov process with the two possible states denoted as: y_l and y_h with $y_h > y_l \geq 0$. Hence, consistently with our general setting, we now assume that there are two assets in the economy: a risk free asset and one state-contingent asset, that we assume repays only when the low realization of income, y_l, realizes. In this setting, the assumed bound for the state contingent asset is $a \geq 0$, which rules out possible gambling incentive on portfolio decisions, as our goal is to understand the implication of participation costs on insurance behavior.

4.1 Individual Household's Participation Decision

Let us first formally describe the problem faced by an individual households. The main difference of our setting with respect to Aiyagari (1994) is the possibility to access state-contingent insurance. We describe this choice and how wealth determines it.

The participation choice. Which type of insurance does the agent choose? In this section we demonstrate that this decision is non-monotonic in the individual level of wealth. Denoting individual agents' wealth by $W = wy + B + 1_{y=y_l}a$, the contingent asset market participation choice follows from comparing the indirect utility when participating in the contingent asset market. The utility when participating in the asset market, denoted as U^P is:

$$\begin{aligned} U^P(W, q, q^f, \kappa) &= u\left(W - \left(qa^P + \kappa\right) - q^f B^P\right) \\ &\quad + \beta\left[\pi(y_h|y)V(B^P, a^P, y_h) + \pi(y_l|y)V(B^P, a^P, y_l)\right], \end{aligned}$$

where a^P and B^P are the optimal decision of the state contingent asset and of the risk free asset. The indirect utility obtained when not participating, and denoted with U^N is:

$$U^N(W, q^f) = u\left(W - q^f B^N\right) + \beta\left[\pi(y_h|y)V(B^N, 0, y_h) + \pi(y_l|y)V(B^N, 0, y_l)\right].$$

where B^N is the optimal decision of the risk free asset when an agent does not participate in the state contingent asset market.

These indirect utilities can be computed using the solution to Problem 2. We leave to Appendix A the formal description of this solution.

The comparison between U^P and U^N pins down a threshold value for the cost that determines the insurance behavior for the agent: given aggregate asset prices and individual level of wealth, $\{W, q, q^f\}$, there exists a threshold value for the fixed participation cost, $\overline{\kappa}$, such that when $\kappa \leq \overline{\kappa}(W, q, q^f)$, the household participates in the contingent asset market, $\delta = 1$, and does not participate otherwise, $\delta = 0$ (see Appendix A for a formal proof of this point).

The relationship between the threshold cost value, $\overline{\kappa}$, and individual wealth generates the following non-monotonic insurance participation behavior, along the lines of Proposition 2:

Proposition 3. *(Non-monotone participation) When households' preferences feature decreasing absolute prudence, there exist two threshold values for wealth,* $\underline{W}(\kappa, q, q^f)$ *and* $\overline{W}(\kappa, q, q^f)$*, such that:*

- *For any* $W \geq \overline{W}(\kappa, q, q^f)$*, households with wealth* W *do not pay the cost and use only risk-free bonds to smooth consumption.*

- *For any* $\underline{W}(\kappa, q, q^f) \leq W \leq \overline{W}(\kappa, q, q^f)$*, households with wealth* W *pay the cost* κ *and purchase both contingent assets and risk-free bonds.*

- *For any* $-\overline{B} \leq W \leq \underline{W}(\kappa, q, q^f)$*, households with wealth* W *do not pay the cost and use only risk free bonds to smooth consumption, if they are not borrowing-constrained.*

Proof. See Appendix D.3 for the proof. □

As a consequence, depending on their wealth, agents have different abilities to smooth consumption: not at all where they are constrained (since they cannot afford the costly contingent assets and they cannot use risk-free bonds because of the constrain), perfectly when they are middle-class (since they acquire contingent bonds) and, interestingly, only partially when they are very wealthy (since they prefer not to purchase contingent bonds and use only the risk-free bond).

Therefore, the existence of a tradeoff between enjoying the benefit of insurance and paying the cost to access the contingent asset market creates an endogenous heterogeneity for the participation decision across wealth.

Consumption smoothing for richest and poorest households. We pointed out that the richest and poorest households may not participate in the contingent asset market. What are the consequences of this behavior in terms of insurance? Denoting the growth rates of consumption as follows:

$$g_{y_l|y} = \frac{u'(c(B'(B,a,y),a'(B,a,y),y_l))}{u'(c(B,a,y))} \text{ and } g_{y_h|y} = \frac{u'(c(B'(B,a,y),a'(B,a,y),y_h))}{u'(c(B,a,y))},$$

from Proposition 7 we obtain the following Corollary:

Corollary 4. *Participation costs to the contingent asset market leads to full insurance when the cost is paid and therefore the agent has access to contingent assets, but to imperfect insurance when the cost is not paid:*

$$1 = \frac{g^P_{y_l|y}}{g^P_{y_h|y}} \geq \frac{g^N_{y_l|y}}{g^N_{y_h|y}}. \tag{2}$$

When participating, consumption grows at a rate that depends only on the price of the risk-less asset:

$$g^P_{y_l|y} = g^P_{y_h|y} = \left(\frac{\beta}{q^f}\right)^{-1/\sigma},$$

which implies that insured households' consumption decreases (increases) over time when $q^f \geq \beta$ ($q^f \leq \beta$).

When constrained on their risk-free asset position, agents do not purchase contingent assets; hence, they do not completely insure. Conversely, when households participate in the contingent asset market, they equalize next period marginal utilities and are fully insured.

Note that full insurance is about all the possible next-period income realizations, but this does not imply that middle-class agents will be permanently fully-insured. In fact, if equilibrium asset prices are such that the wealth of middle-class households deteriorates, adverse income shocks might cause them to transit into the poorest wealth category (with wealth between 0 and $\underline{W}$). Hence, as it will become clear next session, the existence of the three social classes described in Proposition 3, which means that the wealth thresholds satisfying the following restrictions: (*i*) $\underline{W}>0$, (*ii*) $\underline{W} < \overline{W}$, and (*iii*) $\overline{W}$ is finite, depend on the equilibrium asset prices.

In the end, we have the following implication about the cross-sectional distribution of insurance:

Corollary 5. *Consumption volatility, defined as how consumption of an agent changes across periods, is non-monotone across the three wealth categories: it is highest for borrowing constrained households, it is lowest for insured middle-class households, and it attains an intermediate value for self-insured rich households.*

This result relates to the stylized facts about the heterogenous degree of risk-sharing and consumption smoothing across US households highlighted in Guvenen (2007) and Gervais and Klein (2010). In contrast, the Aiyagari model predicts that insurance is increasing with respect to wealth, since more wealth helps to better smooth income shocks, which become less and less important if compared to capital income.

4.2 General Equilibrium

In this subsection, we characterize the general equilibrium outcome of our model, obtained by taking into account how financial markets clear and how asset prices adjust. Our main result is that, depending on the level of participation costs, there is a continuum of equilibria that range from the complete markets economy to the Aiyagari economy.

This is summarized by the following proposition:

Proposition 6 (Equilibrium)**.** *For a given initial wealth distribution, there exists* $\underline{\kappa}$ *and* $\overline{\kappa} \geq \underline{\kappa}$ *such that, for any* $\kappa \geq 0$*, there exists an equilibrium as follows:*

(i) *Self-insurance equilibrium: for* $\kappa \geq \overline{\kappa}$*, households use only risk-free assets to smooth consumption and* $q^f = \bar{q}^f$*, where* $\bar{q}^f$ *is the interest rate in the Aiyagari economy. In this case, the participation cost economy coincides with the Aiyagari economy.*

(ii) *Partial insurance equilibrium: for* $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$*, some households participate in the contingent asset market while the others purchase only risk-free assets. Asset prices are as follows:* $q^f(\kappa) > \beta$ *and* $q(y)(\kappa) = q^f(\kappa)\pi(y_l|y)$*.*

(iii) *Perfect-insurance equilibrium: for* $\kappa \leq \underline{\kappa}$*, all households participate in the contingent asset market and are fully insured. Asset prices are as follows:* $q^f = \beta$ *and* $q(y) = \beta\pi(y_l|y)$*.*

Proof. See Appendix D.4 for the proof. □

In particular, for large values of the participation cost, $\kappa > \overline{\kappa}$, the unique equilibrium features *self-insurance* as in the Aiyagari model. For costs lower than $\overline{\kappa}$, the equilibrium features insurance: either the one featuring *partial-insurance* (for intermediate values of participation costs, $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$), or the one featuring *perfect-insurance* (for small values of participation costs, $\kappa \leq \underline{\kappa}$).

We now investigate further the implications of these results.

The equilibrium interest rate. A first difference with respect to Aiyagari concerns the equilibrium interest rate. The risk-free rate decreases smoothly from the discount rate in the case of perfect insurance to the Aiyagari economy's value, when increasing participation costs. This captures the smoothed evolution of aggregate partial insurance from perfect insurance to self insurance in the Aiyagari case.

As pointed out in Aiyagari (1994),[13] when households have only risk-free bonds to self-insure against idiosyncratic shocks (*self-insurance equilibrium*), the interest rate paid on these bonds is lower than the interest rate paid when markets are complete.[14] The intuition for this result is simply that high level of interest rates would incentivize households to accumulate an infinite amount of assets, which would allow them to consume infinitely and, of course, to be perfectly insured.

A similar result holds in our proposed *partial-insurance* model, but for an additional reason. If the risk-free rate was equal to the full-insurance case (i.e. $q^f = \beta$), households with an intermediate level of wealth, $\underline{W}(\kappa, q, q^f) \leq W \leq \overline{W}(\kappa, q, q^f)$, would always be perfectly insured because their wealth never deteriorates, since the return on their portfolio would be large enough. Hence, these households would never transit into the region characterized by imperfect insurance. In addition, poor households that starts with a low level of wealth, $0 \leq W \leq \underline{W}(\kappa, q, q^f)$, would eventually transit into the perfect-insurance region after receiving a series of positive income shocks. Hence, also those households would be fully insured in the long-run. Finally, rich households with wealth, $W \geq \overline{W}(\kappa, q, q^f)$, either would accumulate an infinitely large quantity of wealth given the high-return on the risk-free assets (as in Aiyagari (1994)) or they would transit into the perfect-insured region after being subject to a series of negative income shocks. Either way, however, they will be obviously perfectly insured.

As a result, if $q^f = \beta$ the unique stationary distribution would feature only perfectly-insured households.[15] For partial insurance equilibria, then we have that $q^f > \beta$, and the distance between q^f and β inversely relates to the amount of contingent insurance purchased by agents.

Initial conditions. Another difference of our setting with respect to Aiyagari (1994) regards how the steady-state equilibrium depends on initial conditions. In the Aiyagari model, the ergodic distribution is independent from the initial wealth distribution but

[13]See also Huggett (1993).

[14]Similarly, Bewley (1980) finds that the optimal rate of inflation should be a little bit higher than the inverse of the discount rate.

[15]This would not be robust to the introduction of aggregate shocks or to idiosyncratic wealth shocks, as, for example, in Blanchard (1985) in which households die according to some Poisson process and other appear with a lower level of wealth.

only depends on the income distribution, technology and preferences' parameters. In contrast, the partial insurance equilibria that characterize our setting inherit some of the dependence on the initial wealth distribution that one can find in complete markets settings, as pointed out in Caselli and Ventura (2000). This dependence is closely linked to the thresholds values of wealth that characterize the insurance region. In fact, recall that, from Corollary 3, for any participation cost there are two associated threshold levels of wealth that pin down the region of wealth associated with participating to the contingent asset market. Now, assume that for a given cost, κ, the initial wealth distribution is all included in the the support $[\underline{W}(\kappa), \overline{W}(\kappa)]$; in this case, all agents participate in the insurance market and therefore they are fully insured. For that level of cost, then, the equilibrium features perfect insurance and, as in Caselli and Ventura (2000) and Chatterjee (1994), the wealth distribution is self-perpetuating. Clearly, for the same level of cost, an economy characterized by an initial wealth distribution W^0 that instead is not contained in the interval $[\underline{W}(\kappa), \overline{W}(\kappa)]$ will feature uninsured agents (the poor with wealth lower than $\underline{W}(\kappa)$, and the rich with wealth higher than $\overline{W}(\kappa)$). In this case, the wealth distribution will converge to a stationary distribution characterized by partial-insurance. As a result, the initial distribution matters for the type of equilibrium achieved in the model.

Partial-insurance equilibria then constitute a continuum of economies between perfect insurance/complete markets and the Aiyagari economy where agents only rely on self-insurance. Along this continuum, the aggregate degree of contingent insurance varies smoothly and this translates into a continuum of risk-free rates. Yet, the cross-sectional distribution of insurance follows a non-trivial pattern, featuring a non-monotone structure, where only middle-class agents are (within-period) perfect insured.

5 Quantitative Implications of Partial-Insurance

In the previous section we have shown that participation costs potentially imply the existence of three categories of households: uninsured and poor, perfectly-insured and middle-class, and self-insured and rich. In this section we quantify how the coexistence of the latter two categories in a *partial-insurance* equilibrium affects the model implications for inequality, wealth concentration, and partial insurance rate.

5.1 Participation costs and the Wealth Distribution

Inequality. How does the existence of participation costs in contingent asset markets affect the wealth distribution? The answer to this question depends on the interaction between participation costs and income risk. For intermediate levels of participation costs that allow for a *partial-insurance equilibrium* to exist, two forces operate in different portions of the wealth distribution. On the one hand, perfectly insured (middle-class) households do not have any incentive to accumulate more assets and, as the risk-less interest rate is lower than in the complete market model, they even progressively consume their wealth. This force pulls the central part of the distribution to the left, compared to the standard Aiyagari model. On the other hand, self-insured richer households benefit from real interest rates that are higher than in the incomplete market model and they accumulate more wealth in comparison with the standard Aiyagari model. This force pushes the right tail of the distribution to the right, compared to that model. Together, these two forces contribute to skew the wealth distribution and lead to large wealth inequality.

To quantitatively investigate these forces, we perform two exercises. First, we show that the endogenous partial-insurance equilibrium implies a much larger wealth Gini coefficient than the self-insurance economy when the exogenous income process is calibrated to realistic values as already used in the literature. Then, we isolate the effects that lead to this larger wealth inequality.

We consider a calibration close to the unemployment economy as in Davila et al. (2012). The utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma = 2$. The discount factor is set at $\beta = 0.96$, so that the annual interest rate is close to 4 percent. The share of capital in the production function is set at $\alpha = 0.36$ and the depreciation rate at 0.08. The only difference with the standard calibration is that we allow for a third state for the income process: $y \in \{0.01, 1, 1.1\}$ but this third state is relatively unlikely so that the income process is very close to the original unemployment economy. The assumed transition matrix is $\pi = \{0.62, 0.38, 0; 0.0199, 0.98, 0.0001; 0, 0.5, 0.5\}$. There are three important comments related to the calibration of the income process. First, our setting delivers the same unconditional moments for the labor market as targeted in Davila et al. (2012), namely a 5 percent unemployment rate and an average unemployment duration of 2.6 years. Second, the inclusion of the third income state assures that the process has enough income variation to guarantee a positive upward social mobility, which is a necessary condition of the existence of a steady-state wealth distribution that features both perfectly-insured and partially-insured agents in presence of intermediate levels of participation costs, as pointed out in the previous section. Also, the inclusion of the third income state allows us to isolate the role of income dispersion in generating upward social mobility by simply varying the magnitude of the income in the third state, leaving all the other entries fixed, as it will be clear in the next section. Finally, the entries of the third row of the transition matrix, which determines the probability to stay in the third state and to transit into the second or first state, are arbitrary calibrated to $[0, 0.5, 0.5]$, but our results are not affected by different choices of these probabilities, as long as the third-state is not absorbing.

We simulate this economy for three different levels of participation cost: a high cost so that the economy is characterized by the *self-insurance equilibrium*, as in the Aiyagari model, an intermediate cost, equal to 0.15, so that the economy is characterized by the *partial-insurance equilibrium* and it implies a wealth Gini index similar to the one observed in the data, and a zero-cost, so that the economy is characterized by the *perfect-insurance equilibrium* (complete markets *de facto*). Table 1 summarizes the main statistics for the three economies.

	High Cost Self-insurance Eq.	Intermediate Cost Partial-Insurance Eq.	No Cost Perfect-Insurance Eq.
Cost/Income	> 0.25	0.15	0
Interest rate (%)	3.244	4.148	4.167
Aggregate assets	3.202	2.963	2.959
Wealth Gini	0.121	0.932	As Assumed

Table 1 – Steady state for the unemployment economy

In the perfect insurance equilibrium (third column), in which participation costs are absent, agents fully insure against idiosyncratic shocks. In this case, no endogenous inequality emerges since the assumed initial wealth distribution is self-perpetuating. Interestingly, there are important differences between the case of the self-insurance equilibrium (first column) and the partial-insurance equilibrium (second column). In the latter, a large mass of agents are trapped with low levels of wealth as they choose to get fully insured. Because interest rates are low enough (lower than the inverse of the discount factor) their wealth deteriorates. In contrast, income fluctuations allow poorer uninsured agents to "jump" above the insurance area when receiving a positive income shock, at which point they become rich and optimally decide to buffer idiosyncratic shocks by accumulating large stocks of assets. As a result, the wealth inequality of the partial-insurance economy is much larger than the wealth inequality of the self-insurance model.[16]

Furthermore, the accumulation of assets by rich agents in the partial insurance equilibrium is even amplified compared to the Aiyagari (1994) model by the larger real interest rate. In fact, in the *partial-insurance equilibrium*, there are lower downward pressures on interest rates than in the incomplete market model because of the existence of households that participate in the contingent markets and that, therefore, have no willingness

[16]Since the initial distribution we have assumed does not entirely lie in the insurance region $(\underline{W}, \overline{W})$, the statistics of the partial-insurance equilibrium refer to the equilibrium stationary distribution.

to accumulate wealth. Notice, however, that albeit the partial-insurance model produces large levels of wealth inequalities, in equilibrium, the interest rate remains lower than in the complete market economy (perfect-insurance). Yet, in contrast to Piketty (2014), in our explanation of inequality, the level of interest rate does not play a central role, but only an amplifying one; wealth is mainly driven by the households' individual *willingness* to accumulate assets, which depends on their insurance choices.

Finally, when participation costs become sufficiently high (first column), no agents purchase insurance anymore, and the economy reverts to the self-insurance equilibrium. Interest rates are lower due to a larger precautionary demand for risk-less assets and there is no discontinuity anymore in the forces that drive wealth accumulation between middle-class and rich agents. As a consequence, the *self-insurance equilibrium* is characterized by a low level of wealth inequality, a result that is well-know in the literature.

The Interest Rate Effect. We can isolate the effect of the higher interest rate and of the insurance area on wealth inequality for the partial-insurance model by running the following exercise. Let us consider first the wealth Gini index resulting from the equilibrium in the economy when participation costs are high, which results in an interest rate of 3.244 percent, and which is equal to 0.121, as displayed in the second row of Table 2. Keeping the same level of cost fixed, we now increase the level of the the interest rate to be equal to the one we obtained in the economy with intermediate cost, 4.148 percent. Obviously, this should be thought as a partial-equilibrium exercise, since that level of interest rate does not clear the capital market when participation costs are high. Nevertheless, the first entry of Table 2 shows that the interest rate effect slightly increases wealth inequality to 0.210, since it allows rich people that have accumulated assets to become even richer, but also that the interest rate effect, alone, is still rather small. Instead, keeping now the same interest rate and decreasing the level of participation costs, thus transiting in the partial-insurance equilibrium, leads to a large wealth inequality, 0.932. This means that the main driver of the skewness of the wealth distribution in our partial-insurance model is the divergent levels of assets among perfectly-insured middle class agents and self-insured rich agents.

The Insurance Trap and the Upper Tail of the Wealth Distribution. We now investigate how the possibility of acquiring insurance leads to higher inequality and to the presence of an insurance trap. First in Figure 1 we plot the Lorenz curve for the wealth distribution in the the self-insurance equilibrium (solid line) and the partial-insurance equilibrium (dashed line) using the benchmark unemployment calibration as described above. As one can observe, the insurance region creates some bunching of households

	High Cost Self-Insurance Eq.	Intermediate Cost Partial-InsuranceEq.
Cost/Income	> 0.25	0.15
Interest rate= 4.148 (%)	0.210	**0.932**
Interest rate= 3.244 (%)	**0.121**	

Table 2 – Wealth Inequality, Interest rates, and Insurance

Note: In bold we report the wealth Gini indices in general equilibrium. The non-bold entry is instead the wealth Gini index obtained in the partial equilibrium exercise that isolates the interest-rate effect on inequality.

at the bottom of the insurance region: households do not have an incentive to accumulate more assets in that region, reducing their upward mobility. The lower incentive to accumulate wealth, despite the higher real interest that prevails in the partial-insurance equilibrium, leads agents in the insurance region to hold less wealth as a share of total wealth compare with the self-insurance equilibrium. Again, this can be illustrated in the case of the simple unemployment economy: Table 3 reports the percentile of agents below, within, and above the insurance region in the partial insurance equilibrium. A small fraction of households (4.4 percent) cannot afford insurance, whereas a large fraction of households (85.6 percent) are perfectly insured. Also, the top 11 percent of households are wealthy enough that they prefer to self-insurance. How is wealth distributed across these three different set of households? As the second row of the table displays, the vast majority of overall wealth (96 percent) is held by the richest households, which are above the insurance region. Keeping fixed the percentiles, we can compute how wealth is distributed in the self-insurance equilibrium, as reported in the third row of the table. It is clear that in the low inequalitarian self-insurance equilibrium most of wealth is instead concentrated in the ends of the central part of the wealth distribution.

	Below insurance	Insurance	Above Insurance
Percentile	0 - 4.4%	4.4 - 89%	89 - 100%
Share of wealth in partial-insurance eq.	1.1%	2.9%	96.0%
Share of wealth in self-insurance eq.	1.8%	84%	14.2%

Table 3 – Shares of total wealth per percentiles

We can now also zoom how wealth is concentrated in the top 10 percent of the distribution in the two equilibria, as displayed in Table 4. Partial insurance generated by

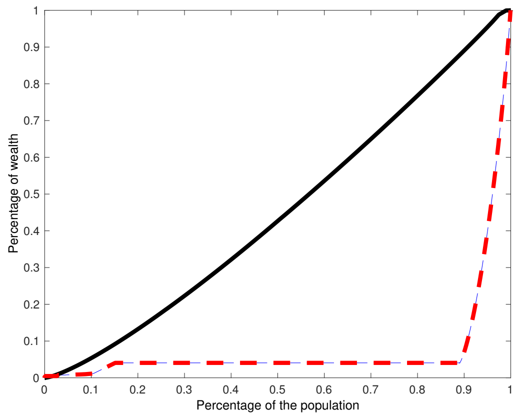

Figure 1 – Lorenz Curves

Note: This graph plots the Lorenz curve for the unemployment economy in the self insurance equilibrium (solid black line) and in the partial-insurance equilibrium (red dashed line).

an intermediate level of participation costs increases the share of wealth for the Top 1% with respect to the standard incomplete market model (columns labelled *High Cost*), although this share remains smaller than in the data. In contrast, for the top 5% or even more for the top 10%, an intermediate level of participation costs, which implies a *partial-insurance equilibrium*, leads to very high wealth concentration with respect to the rest of the population.

	Model		Data
	High Cost	Interm. Cost	
	Self-Insur.	*Part.-Insur.*	
Cost/Income	-	0.15	
Gini	.11	.93	.85
Top 1%	2.0%	15.6%	34.1%
Top 5%	6.6%	60.4%	60.9%
Top 10%	12.2%	93.1%	74.4%

Table 4 – Share of Wealth held by percentile

Note: The model columns refer to the model calibrated to match unemployment moments and used in Davila et al. (2012). *Self-Insur.* indicates the self-insurance equilibrium; *Part.-Insur.* indicates the partial-insurance equilibrium. Data values come from Quadrini and Rios-Rull (2014), Table 14.6.

The Inequality Accelerator. An important condition for the higher level of inequality due to participation to the insurance market is that the stationary *partial-insurance equilibrium* exhibit a non-zero fraction of self-insured rich households. A necessary condition for this to happen is that the economy is subject to large-enough income risk. Intuitively, self insured rich households still face negative income shocks. As the real interest rate is still below the discount rate in equilibrium ($q^f \geq \beta$), these households' wealth can potentially fall below the correspondent upper participation threshold, $\overline{W}$. Because of this existing downward social mobility force and because middle-class agents are perfectly insured, so that positive income shocks do not affect their next-period wealth, then it is necessary that some poorer non-perfectly insured households can possibly jump above the insurance area to obtain a stationary *partial-insurance equilibrium* featuring self-insured rich households.

We now conduct a comparative static exercise to illustrate how different levels of income inequality may lead or not to the existence of a group of rich non-insured households. As this creates a potential non-linear relationship between income and wealth inequality, we refer to this mechanism as the *inequality accelerator* effect. We consider two income processes: the same income process as in the previous paragraph: $y \in \{.01, 1, 1.1\}$ associated with

$$\pi = \{0.62, 0.38, 0; 0.0199, 0.98, 0.0001; 0, 0.5, 0.5\},$$

and a slightly different one: $y \in \{0.01, 1, 1.05\}$ associated with the same transition matrix. Notice that the second process is characterized by a smaller income dispersion across the states, but since the third state is unlikely the overall income Gini inequality of the two processes is almost identical and equal to 0.097. Hence, in Table 5, which reports the equilibrium wealth Gini index resulting from both income processes, we label the first process as the *High Income Inequality* and the second process as the *Low Income Inequality*.

	High Cost	Intermediate Cost	No Cost
Cost/Income	>0.25	0.15	0
		Wealth Gini Index	
High Income Risk	0.121 *Self-Insur.*	0.932 *Part.-Insur.*	*As Assumed* *Perf.-Insur.*
Low Income Risk	0.110 *Self-Insur.*	*As Assumed* *Perf.-Insur.*	*As Assumed* *Perf.-Insur.*
Davila et al. (2012)	0.108 *Self-Insur.*	-	-

Table 5 – Steady state for the unemployment economy for different income risk

Note: We consider two income processes. The high income risk process has as entries $y \in \{.01, 1, 1.1\}$.The low income risk process has as entries $y \in \{.01, 1, 1.05\}$, keeping the same transition matrix. The table reports the wealth Gini indexes for three different levels of cost (0.25, 0.15, 0), and the type of equilibrium associated with each combination of cost/income risk. *Self-Insur.* indicates the self-insurance equilibrium; *Part.-Insur.* indicates the partial-insurance equilibrium; *Perf.-Insur.* indicates the perfect-insurance equilibrium;

Let's analyze first the case with no-costs (third column). In that case, all the agents participate in the state-contingent market and are fully insured, the economy is characterized by the *perfect-insurance* equilibrium and, therefore, the size of income risk is irrelevant for inequality, since the equilibrium wealth distribution is equal to the assumed initial one. On the contrary, the level of income-risk largely affects the resulting equilibrium in presence of intermediate costs (second column): if positive income shocks are too small (second row), poorer non-insured agents cannot "jump" above the insurance area; in this case there are not rich agents in equilibrium, everyone will be perfectly insured, and, therefore, for that intermediate level of cost the economy is in the *perfect-insurance equilibrium.* In contrast, when income fluctuations become slightly larger (first row), positive income shocks allow agents to "jump" above the insurance area. Then, these households continue to accumulate assets for self-insurance purpose. Hence, the same intermediate level of cost implies a *partial-insurance equilibrium* with a high income-risk process. Another way to interpret these results is pointing out that the threshold level of cost $\underline{\kappa}$ that separates the *perfect-insurance equilibrium* and the *partial-insurance equilibrium* as stated in Proposition 6, is a negative function of the exogenous income risk. By lowering the degree of income risk in the economy, it takes a larger level of cost to move from a *perfect-insurance equilibrium* to a *partial-insurance equilibrium*; the intermediate cost in the second column of Table 5 is above the threshold associated with the high income risk process and below the threshold associated with the low income risk process.

Finally, notice that the inclusion of the third income state with respect to the calibration in Davila et al. (2012), as well as considering our high income-risk or low income-risk processes, does not affect *per-se* wealth inequality, since in the *self-insurance equilibrium* (first column) the two three-state income processes leads to a basically identical very low wealth Gini coefficient to the one reported by Davila et al. (2012), which consider a two-state income process that leads to the same unconditional unemployment moments.

Therefore, it is important to remark that the rationale behind the large wealth inequality achieved in our setting differs from the ones in Castaneda et al. (2003). In fact, in their incomplete market model the large wealth inequality is solely driven by the very large income dispersion (income Gini index equal to 0.600), which translates into a large income risk for the top-earners. In contrast, in our setting, a much smaller degree of income fluctuations (income Gini index equal to 0.097) is able to trigger a sizable wealth inequality not only through the much weaker channel of income risk for the top-earner, but, above all, through the different insurance incentives across the wealth distribution and asset prices. To summarize, the economy characterized by intermediate levels of participation costs requires only a certain (small) degree of income inequality to trigger the large amplification from income inequality to wealth inequality mainly driven by the non-monotone willingness to insure across the wealth distribution and its implications on asset prices.

5.2 Participation Costs and Partial Insurance

As discussed in the previous section, the joint presence of insured and self-insured households increases the level of wealth inequality. Obviously, the coexistence of fully insured and self-insured households implies a certain degree of aggregate insurance in the economy. In this section we explore the relationship between participation cost, wealth inequality, and degree of partial insurance.

Let us denote the equilibrium share of insured agents by θ. Hence, θ directly represents the fraction of insured households. In addition, applying the law of large numbers and noticing that that each agent's income is independently distributed, θ also represents the average share of individual income that is insured, or, equivalently, the degree of partial insurance, as defined in Guvenen and Smith (2014). We compute the share of insured households for our benchmark unemployment economy as well as for a different calibration of the model, which is the original Aiyagari calibrations.[17] Notice that the different calibrations require two slightly different sizes of the participation cost for the model to be able to generate a wealth Gini of around 0.9, as the two different calibrations lead to different patterns of risk and, thus, demand for insurance. Table 6 reports the obtained results.

		Unemployment	Aiyagari (1994)
High cost	Gini	0.11	0.42
	θ (%)	0	0
Intermediate cost	Gini	0.93	0.96
	θ (%)	84.6	31.6
	Cost/Income	0.15	0.22

Table 6 – Degree of partial insurance

The top-panel of the table displays the resulting characteristics in case of high participation costs. In this setting, there are no households that enter in the contingent asset market and, therefore, the fraction of insured agents, or equivalently the fraction of total insured income, is zero. The two calibrations imply rather low level of inequality, as indi-

[17]The calibration for the Aiyagari (1994) model is as in p.19 of Davila et al. (2012). The coefficient of relative risk aversion in the CRRA utility function is set to 2. The discount factor is set to 0.96. The capital share is equal to 0.36. The three state process for income is $y = \{0.78, 1.00, 1.27\}$. The transition matrix is $\pi = \{0.66, 0.17.0.27; 0.28, 0.44, 0.28; 0.07, 0.27, 0.66\}$.

cated by a wealth Gini index of 0.11 for the unemployment economy and of 0.42 for the Aiyagari (1994) economy. The bottom-panel of the table reports the same equilibrium statistics in the presence of intermediate levels of participation costs. When participation costs are not excessively high, the degree of partial insurance, as well as the wealth Gini coefficient, increases. Yet, the relationship between the degree of partial insurance and the increased level of inequality is not trivial and is calibration-dependent: even a small share of partial insurance, which corresponds to a small share of participation, around 30 percent, as in the case of Aiyagari (1994) economy, is able to skew the wealth distribution to provide with a Gini index similar to one observed in the U.S. data. In this case, even with a minority of insured households, the equilibrium level of interest rate and the social mobility effect described in the previous section give rich agents large incentives to accumulate wealth. Interestingly, the unemployment economy implies a similar level of inequality with a much larger participation rate, around 80 percent. In this setting, the properties of the income process are such that even a small fraction of self-insured household has strong incentives to accumulate a large amount of wealth.

Our definition of partial insurance can be linked to the one introduced in Guvenen and Smith (2014). However, whereas their form of partial insurance is on the intensive margin - agents can insure a fraction of their income, in our setting partial insurance is on the extensive margin - agents can be insured or not. In their empirical work, Guvenen and Smith (2014) estimates the fraction of partial insurance around 45 percent. In this section we showed how the existence of participation costs, which leads to partial insurance, can generated realistic level of wealth inequality together with degree of partial insurance both above and below their estimated partial insurance level.

6 Further extensions and discussion

Downward and upward insurance. Several examples in the literature underscore the comparatively lower levels of insurance coverage for poorer households than for the rest of the population.[18] In those studies, lack of insurance concerns *downward shocks*, which is future negative income shocks. This differs from borrowing constraints that limit the ability of insurance against *upward shocks*, which is positive future income shocks. Of course, as poor households are also likely to be borrowing constrained, they are also uninsured upward.

[18] Brown and Finkelstein (2007) documents lower private long-term care insurance coverage for poorer households. An additional example is provided by Cole et al. (2009) when studying insurance behavior of Indian farmers. Murdoch (1995) studies how farmers in India choose to lower their average income against lower volatility. Cole and Shastry (2009) show in a different context that education is also a determinant of insurance decisions, providing a non-monetary interpretation of participation costs.

Our participation cost-model is able to reproduce such lack of both downward and upward insurance for poor households. In contrast, the one-sided no-commitment model (see Thomas and Worrall (1988) as an example) fails to reproduce the downward non-insurance: short-selling or borrowing constraints only prevent households from borrowing against future revenue and not from accumulating assets for insuring against lower future income. In comparison, the standard Aiyagari (1994) model is compatible with the absence of downward non-insurance, but it cannot account for endogenous insurance decisions as it simply rules out insurance contracts.

Interpreting participation costs. Our baseline interpretation of participation costs is a monetary one. These monetary costs arise from financial or insurance intermediaries, possibly related to sunk costs due to an intermediaries' production functions or to screening costs, when agents have to signal their type by willing to pay the fixed costs.[19] Other interpretations include cognitive costs or shopping-costs: selecting insurance requires time and effort. All these interpretations imply paying the fixed cost *ex ante*. Another alternative form of fixed cost faced by households surfaces when collecting insurance payments when bad shocks occur. Collection requires proofs of damage to address the adverse selection problem. Assuming this alternative form of participation cost would not qualitatively change our results: it would also prevent agents from purchasing insurance against small shocks, and would lead to preferences for purchasing insurance only against large shocks. In this situation, as in our setting, poorer households cannot afford to pay the insurance.

Long-term assets. Our theory considers only one-period assets that require paying the participation costs at every period; however, in the real world, one might argue that insurance is a long-term proposition.

In our framework one could introduce a long-term asset that pays nothing as long as good shocks occur but yields a payoff in case of a bad shock. The main difference with respect to a short-term asset is that this long-term asset can be held for several periods, until a bad shock occurs and triggers a payoff to the agent. Our analysis can easily be extended to similar long-term assets under the assumption that the long-term insurance stops after a bad shock. Otherwise, the insurance would be purchased once and for all. More general long-term assets can also be considered, but they have to remain, to some degree, contingent on agents' idiosyncratic shocks.

[19] The exact setting leading to this kind of fixed cost would be a dynamic version of Rothschild and Stiglitz (1976).

7 Concluding remarks

In this paper, we study the *partial-insurance equilibrium* that characterizes an economy with participation cost in state-contingent asset markets. In this setting households' degree of insurance depends on their wealth. In fact, under decreasing absolute prudence, the *partial-insurance equilibrium* is characterized by a set of poor households that are not able to obtain any insurance, by a set of middle-class household that actively participate in the contingent asset market and, hence, are fully insured, and, interestingly, by a set of rich households that prefer to self insure by accumulating a large stock of the risk-free assets.

This non-monotonic relationship between degree of insurance and wealth leads to important implications about social mobility, wealth inequality, and wealth concentration. Specifically, when participation costs reduce from an arbitrary large value, such that the economy is equivalent to a *self-insurance equilibrium*, to intermediate values, such that the economy turns into a *partial-insurance equilibrium*, wealth inequality dramatically increases. With intermediate value of participation costs, our model can predict a level of wealth inequality similar to the one observed in the U.S. data (Gini index equal to 0.93). Our quantitative section shed lights on the importance of partial insurance to increase inequality. In fact, perfectly insured middle-class households do not have incentive to accumulate assets for insurance purposes, while the richest ones do. This feature skews upward social mobility so that middle-class agents are less likely than richest agents to increase their wealth and, as a result, the upper tail of the wealth distribution thickens in presence of intermediate levels of participation costs. Our model can predict that 60 percent of total wealth is held by the top 5 wealth percentile, a feature that is close to the one observed in the data and much larger than the one implied by the standard Aiyagari (1994) model (6.6 percent). In addition, a general equilibrium effect reinforce the skewness of the wealth distribution since in a model with partial-insurance the interest rate is larger than in Aiyagari (1994)'s model. Our paper has, then, important implications for households' insurance decisions, asset prices, social mobility, and inequality.

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A Additional elements on household decision - Not for publication

In this setting the first order conditions for Problem 2 yield:

$$
\begin{aligned}
&V_B(B,a,y) = u'(wy + B + 1_{y=y_l}a - \delta\,(qa' + \kappa) - q^f B'),\\
&V_a(B,a,y) = 1_{y=y_l}u'(wy + B + 1_{y=y_l}a - \delta\,(qa' + \kappa) - q^f B'),\\
&q^f u'(wy + B + 1_{y=y_l}a - \delta\,(qa' + \kappa) - q^f B') = \beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_B(B', a', y) + \gamma,\\
&\delta q u'(wy + B + 1_{y=y_l}a - \delta\,(qa' + \kappa) - q^f B') = \delta\beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_a(B', a', y),
\end{aligned}
$$

where γ is the Lagrange multiplier associated with the borrowing constraint $B' \geq -\overline{B}$.

When the agent decides to participate in the contingent asset market, i.e. $\delta = 1$, these equations define a^P and B^P. Similarly, when $\delta = 0$, they define $a^N = 0$ and B^N, where, as before, the superscript P denotes asset holding when participating in the contingent asset market and superscript N when not participating.

Remark. Uninsured agents ($\delta = 0$) purchase only risk-free assets. Their first order conditions are:

$$
\begin{aligned}
&V_B(B,a,y) = u'(wy + B - q^f B'),\\
&u'(wy + B - q^f B') = \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_B(B', 0, y) + \gamma.
\end{aligned}
$$

Hence, uninsured agents solve a similar problem as households in Aiyagari (1994).

Our first result is a no-arbitrage condition easily derived from the first order conditions above and that puts a restriction on asset prices:

Proposition 7 (Asset prices)**.** *Constrained households (for which $\gamma > 0$ in state y) do not purchase contingent assets as long as:*

$$q(y) \geq q^f \pi(y_l|y).$$

When there are unconstrained households ($\gamma = 0$) that participate in the contingent asset market, the following no-arbitrage condition is satisfied:

$$q(y) = q^f \pi(y_l|y).$$

Proof. Manipulating first-order conditions yields:

$$\frac{u'(y_h|y)}{u'(y_l|y)} = \frac{\beta\pi(y_l|y)}{q}\left(\frac{q^f - q}{\beta\pi(y_h|y)} - \frac{\gamma}{u'(y)\beta\pi(y_h|y)}\right).$$

At most, the agents are willing to equalize marginal utilities $u'(y_h|y) = u'(y_l|y)$ and, in addition, the positivity of γ lead to:

$$\frac{\beta\pi(y_l|y)}{q}\,\frac{q^f - q}{\beta\pi(y_h|y)} \geq 1 \text{ or, equivalently } q^f\pi(y_l|y) \geq q.$$

□

The first consequence of this proposition is that there are only two types of portfolio in the economy: either households trade only risk-free assets or they trade both contingent and risk-free assets. Indeed, constrained households' willingness to purchase contingent assets is strictly lower than for unconstrained households. Therefore, when smoothing consumption, the household has a choice between a non-targeted but cheap insurance (by using only risk-free assets) and a targeted but costly insurance (by using both types of assets).

Participation.

Proposition 8. *Given aggregate asset prices and individual level of wealth, $\{W, q, q^f\}$, there exists a threshold value for the fixed participation cost, $\overline{\kappa}$, such that when $\kappa \leq \overline{\kappa}(W, q, q^f)$, the household participates in the contingent asset market, $\delta = 1$, and does not participate otherwise, $\delta = 0$.*

Proof. The choice to participate amounts to comparing $U^P(W, q, q^f, \kappa)$ and $U^N(W, q^f)$. Using the envelope theorem, the derivatives of $\Delta = U^P(W, q, q^f, \kappa) - U^N(W, q^f))$ are:

$$\frac{\partial \Delta}{\partial \kappa} = -u'(W - qa^P - \kappa - q^f B^P) < 0.$$

In addition, when $\kappa = 0$, participation is preferred to non-participation, as, when participating, the household can do as good as when not participating. As a result, there exists then $\overline{\kappa}$ such that households accept to pay the cost κ if and only if $\kappa \leq \overline{\kappa}$. □

B Value function and recursive formulation - Not for publication

The following proposition establishes the existence and the uniqueness of the value function solving Problem 2.

Proposition 9. *The value function V exists and is unique.*

Moreover, the value function V can be obtained by iterations: for any initial value $V' \in \Omega$ and defining the sequence, $V_n = T^n V'$, V_n converges to V.

Proof. This proof extends the proof of Stokey et al. (1989) for discrete variables. Recall that the value function satisfies:

$$V(B, \{a\}, y) = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \begin{bmatrix} u(C) \\ + \beta \sum_{y'} \pi(y'|y) V(B', \{a'\}, y') \end{bmatrix}$$

Defining T as:

$$TV = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{y'} \pi(y'|y) V(B', \{a'\}, y') \right]$$

it is easy to show that T satisfies Blackwell's conditions. First T is monotonic. For $W \leq V$, we have that :

$$\begin{aligned} TW &= \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{y'} \pi(y'|y) W(B', \{a'\}, y') \right] \\ &\leq \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{y'} \pi(y'|y) V(B', \{a'\}, y') \right] \\ &= TV \end{aligned}$$

Second T discounts: let Γ be a positive constant:

$$\begin{aligned} T(V+\Gamma) &= \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{y'} \pi(y'|y) \left(V(B', \{a'\}, y') + \Gamma \right) \right] \\ &= \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \left[\Gamma + \sum_{y'} V(B', \{a'\}, y') \right] \right] \\ &= TV + \beta\Gamma \end{aligned}$$

We define $X = \{x = \{B', \{a'\}, y'\}\}$. Ω denotes the set of functions V such that V is continuous with respect to B and a. We need also to prove that:

- Ω with the d_∞ metric is a metric space.
- TV is in the same set as V, which is obvious.

Metric space Let $\{V_n\}$ a Cauchy sequence of Ω. For every $x \in X$, $V_n(x)$ converges to $V(x)$. Let us verify that V is the limit using the d_∞ metric. As $\{V_n\}$ a Cauchy sequence: for some $\epsilon > 0$ and for some $x \in X$, there exists n such that for every p and q satisfying $q \geq p > n$, $|V_p(x), V_q(x)| < \epsilon$. Taking the limit of this expression with respect to q, we obtain that $|V_p(x), V(x)| < \epsilon$. As this is true for every $x \in X$, this implies that $d_\infty(V_p, V)$ converges to 0, which means that V_n converges to V.

Conclusion The requirements of the Contraction Mapping theorem are satisfied. There exists an unique $V \in \Omega$ such that $TV = V$. Furthermore, for any $V' \in \Omega$ and defining $V_1 = TV'$ and, more generally, $V_n = T^n V'$, V_n converges to V. This makes possible iterations on the value function as usual. □

The connexion between being solution to Problem 1 and to Problem 2 easily obtains from standard results, at least in the case of bounded utility function (see Stokey et al., 1989). Indeed, in that case, the discrete participation choice does not prevent $\lim_{n->\infty} \sum_{t=0}^{n} \beta^t u(c_t)$ to exist (and be finite), which allows to use Theorems 4.2 to 4.5 in chapter 4, thus guaranteeing the equality between the two solutions. When using unbounded utility functions, this result is more difficult to obtain, but it is not related to the discrete decision.

C Multiple states and order of insurance - Not for publication

Having analyzed the market participation with two states, we now study the participation decisions for an arbitrary number of states. Buying insurance contingent to one particular state decreases one agent's wealth and, hence, modifies his willingness to participate in another contingent asset market. As a results, agents face a trade-off when insuring against multiple states. In this section, we first illustrate the interaction between insurance against different states and, second, we show that households choose insurance following a sequential order.

The effects of initial wealth on multiple insurances Given the strict relationship between asset market participation decision and agents' wealth as shown in Corollary 3, we now focus on the agents with an intermediate level of wealth. In particular, we assume that there are gains from participating in each contingent asset market k:

$$\begin{aligned} & u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y) \\ & < u(W - q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_m \pi(y_m|y) V\left(B^P, \{0,..,a_k^P,..0\}, y_l\right). \end{aligned}$$

However, it is not a foregone conclusion that the agent can afford to access to every asset market, as we may have:

$$\begin{aligned} & u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y) \\ & > u(W - \sum_k q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_k \pi(y_k|y) V\left(B^P, \{a^P\}_k, y_k\right). \end{aligned}$$

If this condition is satisfied, the household prefers not to buy insurance against every state of the nature. Intuitively, buying insurance against one state decreases the resources available to buy insurance against another state.

Sequential decision When a household is able to participate in contingent asset markets only to a limited degree, she chooses sequentially to buy insurance against different states. Intuitively, we will show that the utility obtained by insurance against one state is proportional to the distance between the threshold cost and the actual associated cost of insurance for that state. Hence, that distance provides a criterion for ranking different assets. The first state against which the agents will insure is the one where the distance between the actual participation cost and the threshold is maximized. Moreover, by insuring against more and more states, agents decrease their wealth because of participation costs. When their wealth is low enough, agents stop buying further insurance.

In order to rigorously define the sequential decision, we define two concepts: the set of feasible insurance, and a choice of insurance.

Definition 3. The set of feasible insurance $F(y)$ is a subset of Y, such that for every $k \in Y$, gains with respect to the completely non-insurance case are positive:

$$\begin{aligned} & u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y) \\ & < u(W - q(k,y)a(k) - \kappa(k,y)) + \beta \sum_m \pi(y_m|y) V\left(B^P, \{0,..,a_k^P,..0\}, y_m\right). \end{aligned}$$

i.e. participation in the asset market contingent to the state k is preferred to autarky.

A choice of insurance at period t is a subset $I(y)$ of $F(y)$.

The recursive problem for household i writes:

$$\max_{I(y)\subset F(y)} \max_{\{a(h')\}} \left[\begin{array}{r} u\left(W - \sum_k \delta_{k\in I(y)}(q(k,y)a(k) + \kappa(k,y))\right) \\ +\beta \sum_l \pi(y_l|y) V\left(B^P, \{a_l^P\}, y_l\right) \end{array} \right]. \tag{3}$$

The following Proposition characterizes the solution of the sequential insurance problem faced by the agents, and states the analogy between gains from accessing the asset market and distance between participation costs and the threshold costs.

Proposition 10 (Pecking order of access to markets)**.** *The ordering of asset market participations of households follows the gains with respect to non-participation:*

$$\begin{aligned} & u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y) \\ & < u(W - q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_l \pi(y_l|y) V\left(B^P, \{0, .., a_k^P, ..0\}, y_l\right). \end{aligned}$$

These gains map with the same order as the distance between costs ($\kappa(k,y)$) and thresholds ($\overline{\kappa}(k,y)$): the higher the gains, the greater the difference: $\overline{\kappa}(k,y) - \kappa(k,y)$.

Proof. Program (3) is:

$$\max_{I(y)\subset F(y)} \left\{ \max_{\{a(k)\},B} \left[\begin{array}{l} u\left(W - \sum_{k\in I(y)} q(k,y)a(k) + \kappa(k,y) - q^f B\right) \\ + \sum_l \pi(y_l|y)\beta V(B, \{a(k)\}, y_l) \end{array} \right] \right\}$$

Consider now a sequential choice following this iterative algorithm:

- Initial condition: set of possible choices: $S = F(y) \subset Y$, list: $L = \emptyset$
- Iteration:
  - y_k is the state in S which gives the highest gain compared to non-participation.
  - $L = L \cup y_k$ and $S = S - y_k$

This algorithm stops as S is a finite set.

As this algorithm yields a sequence L, we define by $I(y)$ the set of elements of this sequence and now, we have to prove that this set solves optimization (3). Consider a state h^1 in $F(y) - I(y)$ and a state h^2 in $I(y)$. Using lemma 11, we have the result. □

Lemma 11 (Local property)**.** *$I(y)$ maximizes utility if and only if $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility for any $h^1 \in I(y)$ and $h^2 \in F(y) - I(y)$.*

Proof. First we show the implication from left to right. This is trivial as $I(y)$ maximizes utility contradicts the proposition that there exists a h^2 in $F(y) - I(y)$ and there exists a h^1 in $I(y)$ such that $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility.

Second we show the implication from right to left. Suppose that $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility for any $h^1 inI(y)$ and $h^2 \in F(y) - I(y)$. We proceed by contradiction by supposing then that $I(y)$ does not maximize utility and that there exits I' which maximizes utility. I' cannot be a subset of $I(y)$

and $I(y)$ cannot be a subset of I' neither, considering the stopping condition of the iterative algorithm. There exist then h^2 in I' but not in $I(y)$ and h^1 in $I(y)$ but not in I'. It is easy to check that we can get more utility by taking with $I(y)' - \{h^1\} \cup \{h^2\}$ compared with I', which contradicts the fact that I' maximizes utility. □

Two specific cases merit consideration. First, when costs are uniform across states, according to Proposition 10, households become insured against the worst possible or best possible state. They begin with the worst and the best and, progressively, they purchase insurance against less extreme future outcomes. Second, when costs are sufficiently increasing along with income shocks, agents may become insured only against small shocks, not against large income variations, since the latter case involves paying larger participation costs. This situation is consistent with recent research about insurance(Cole et al. (2009)). However, modeling increasing fixed costs would require further micro-foundations that are beyond the scope of this paper.

D Proofs of propositions - Not for publication

D.1 Proof of Proposition 1.

First, notice that the feasibility condition $W > L$ assures that consumption is always is strictly positive. To prove (1) we only need the conditions that $u'(x) > 0$, $u''(x) < 0$. Since the utility function is increasing, $u'(x) > 0$, then we have the following ordering: $u(W - L) < u(W) < u(W + pL/(1 - p))$. Therefore, gain of insurance is positive if:

$$u(W) \geq pu(W - L) + (1 - p)u(W + pL/(1 - p)),$$

which holds as the utility function is concave.

To prove (2), notice that concavity of the utility function implies:

$$\begin{aligned} [u(W) - u(W - L)] &< u'(W - L)L, \\ [u(W) - u(W + pL/(1 - p))] &< -u'(W + pL/(1 - p))pL/(1 - p), \end{aligned}$$

Since $u'(x)$>0 then, for any $W > 0$, we have:

$$0 < G(W, 0) < u'(W - L)L - u'(W + pL/(1 - p))pL/(1 - p),$$

and by the Inada condition, $\lim_{W\to\infty} u'(W - L) = \lim_{W\to\infty} u'(W) = 0$ and therefore $\lim_{W\to\infty} G(W) = 0$.

To prove (3), notice that from equation (1), the effect of wealth on the gain of insurance is given by:

$$\frac{\partial G(W, 0)}{\partial W} = \frac{1}{2}\left(u'(W) - (1 - p)u'\left(W + \frac{pL}{1 - p}\right) - pu'(W - L)\right). \tag{4}$$

we need to show that the right-hand-side of equation (4) is negative. The proof follows the same argument as for proving (1) by using the property of $u''(x) < 0$ to order the points as follows: $u'(W - L) > u'(W) > u'(W + pL/(1 - p))$ and by using the convexity of u', i.e. $u'''(x) > 0$ to prove the inequality.

D.2 Proof of Proposition 2.

The proof of (1) follows three steps. First, we prove that the function $G(W, \kappa)$ has one and only one minimum at a wealth level W^*. Second, we prove that $G(W^*, \kappa) < 0$. Third, we prove that under the condition of the cost, there exists a unique threshold level $\bar{W}$.

Feasibility in each state and time requires that $W > L$ and that $\kappa < 2L$. Suppose that $u'''' < 0$. As $u''' > 0$ and $u'' < 0$, the coefficient of absolute prudence, $P(W) = -\frac{u'''(W)}{u''(W}$, is decreasing in W, as its derivative has the sign of $-u''''u'' - (u''')^2$. Similarly to Kimball (1990b), we define as the *precautionary equivalent premium* the function $\psi(W)$ such as $u'(W - \psi(W)) = \frac{1}{2}u'(W) + \frac{p}{2}u'(W - L) + \frac{1-p}{2}u'(W + pL/(1-p))$. Given the properties of the utility function, $\psi(W)$ is non-negative, strictly decreasing in W and converges to 0 when W goes to ∞ (see Proposition 62 in Gollier (2004)). Hence, $\forall W \in [L, \infty)$, $\psi(W)$ is invertible and $\psi^{-1} \in (0, \psi(L)]$. Notice that by applying the definition of $\psi(L)$ and using the Inada conditions, we have that: $\psi(L) = L$. Finally, note that $G(W, \kappa)$ converges to 0 when W goes to ∞. As a consequence, for all $\kappa \leq 2L$, there exists a unique level of wealth $W^*(\kappa) = \psi^{-1}(\kappa/2)$ such that $u'(W^*(\kappa) - \kappa/2) = \frac{1}{2}u'(W^*(\kappa)) + \frac{p}{2}u'(W^*(\kappa) - L) + \frac{1-p}{2}u'(W^*(\kappa) + pL/(1-p))$; hence, for all $\kappa \leq 2L$, there exists a unique $W^*(\kappa)$ such that $\frac{\partial G(W^*(\kappa),\kappa)}{\partial W} = 0$. As $\psi(W)$ is decreasing, for $W' > W^*(\kappa)$:

$$\begin{aligned}
&u'(W' - \kappa/2) - \frac{1}{2}u'(W') - \frac{p}{2}u'(W' - L) - \frac{1-p}{2}u'(W' + pL/(1-p)) \\
&= u'(W' - \psi(W^*(\kappa))) - \frac{1}{2}u'(W') - \frac{p}{2}u'(W' - L) - \frac{1-p}{2}u'(W' + pL/(1-p)) \\
&\geq u'(W' - \psi(W')) - \frac{1}{2}u'(W') - \frac{p}{2}u'(W' - L) - \frac{1-p}{2}u'(W' + pL/(1-p)) = 0,
\end{aligned}$$

which implies that for $W' > W^*(\kappa)$, $\frac{\partial G(W',\kappa)}{\partial W} > 0$. The same reasoning for $W' < W^*(\kappa)$ implies that for any $W' < W^*(\kappa)$, $\frac{\partial G(W',\kappa)}{\partial W} < 0$. We have proved that $G(W, \kappa)$ has a unique minimum in $W^*(\kappa)$.

As a second step, notice that since $G(W, \kappa)$ admits exactly one minimum $W^*(\kappa)$, is decreasing for any $W < W^*(\kappa)$, is increasing for any $W > W^*(\kappa)$, and converges to 0 when W goes it ∞, then necessarily $G(W^*(\kappa), \kappa) < 0$. Notice that for any $W' > W^*(\kappa)$, then $G(W', \kappa) < 0$. We have proved that the minimum of $G(W, \kappa)$ is negative.

As a third step, let $\hat{\kappa}$ be the value of the cost that solves: $G(L, \hat{\kappa}) = 0$, i.e.:

$$u(L - \frac{\hat{\kappa}}{2}) = \frac{1}{2}\left[u(L) + (1-p)u(L/(1-p)) + pu(0)\right].$$

Since by Proposition 1 $G(L, 0) > 0$, since $G(L, 2L) < 0$, and since obviously $G(L, \kappa)$ is decreasing in κ, then by the intermediate value theorem, $\exists!\ \hat{\kappa}$: $G(L, \hat{\kappa}) = 0$. Then, for any feasible κ such that $\kappa < \hat{\kappa}$, then $G(L, \kappa) > 0$ and $G(W, \kappa)$ reach a negative value at its minimum; hence, by the intermediate value theorem, exists a unique $\overline{W}(\kappa) < W^*(\kappa)$ such that $G(\overline{W}(\kappa), \kappa) = 0$.

The proof of (2) comes easily. First, by Proposition 1, $\forall W > L$, $G(W, 0) > 0$. Hence, $\forall W > L$, $V^P(W, \kappa) > V^N(W)$ and by definition $\mathbb{P}(0) = \{W : W > L\}$. This implies that for all feasible wealth levels, $W > L$, households participate to the insurance market in the absence of participation costs as noticed in Proposition 1. Second, notice that by using the implicit function theorem, $\frac{\partial G(L,\kappa)}{\partial \kappa} < 0$. Hence, $\frac{\partial \overline{W}(\kappa)}{\partial \kappa} < 0$. Therefore, $\forall \kappa_2 > \kappa_1$, $\mathbb{P}(\kappa 2) \subset \mathbb{P}(\kappa_1)$. Finally, as κ increases above $\hat{\kappa}$, $G(L, \kappa) < 0$, and, therefore, $\forall W > L$, $G(W, \kappa) < 0$. In this case $V^P(W, \kappa) < V^N(W)$ and by definition $\mathbb{P}(\kappa) = \emptyset$.

D.3 Proof of Proposition 3.

First, note that there exists W sufficiently small so that $B^P = B^N = -\overline{B}$. In this case, we can show that the agent will not participate in the state contingent market, which means that the following relationship is satisfied:

$$
\begin{aligned}
& u(W-(q(y)a^P+\kappa)+q^f\overline{B})-u(W+q^f\overline{B}) \\
& \leq \beta\left[\begin{array}{r}\pi(y_l|y)\left[(V(-\overline{B},a^P,y_l)-V(-\overline{B},a^P,y_h))-(V(-\overline{B},0,y_l)-V(-\overline{B},0,y_h))\right] \\ +(V(-\overline{B},a^P,y_h)-V(-\overline{B},0,y_h))\end{array}\right] \\
& \leq \beta\pi(y_l|y)\left[(V(-\overline{B},a^P,y_l)-V(-\overline{B},a^P,y_h))-(V(-\overline{B},0,y_l)-V(-\overline{B},0,y_h))\right]. \qquad (5)
\end{aligned}
$$

In fact, the Inada condition implies that when W approaches $-\overline{B}$, a^P tends to 0 and B^P and B^N tend to $-\overline{B}$. Hence, as long $\kappa > 0$, a sufficiently decreasing W implies that $u(W-(q(y)a^P+\kappa)+q^f\overline{B})-u(W+q^f\overline{B})$ goes to $-\infty$, and equation (5) is then verified.

We now prove that also when W large the agent also prefers not to participate. First, notice that $\lim_{W\to\infty} a^P < \infty$, as there is no gain of infinitely accumulating contingent assets, but $\lim_{W\to\infty} B^P = \lim_{W\to\infty} B^P = \infty$. Prudence ($u''' > 0$) and the fact that $qa^P+\kappa \geq 0$ imply that $B^N > B^P$, for all W and $\kappa > 0$. Furthermore, decreasing absolute prudence ($u'''' < 0$) implies that that $B^N - B^P$ decreases with W and converges to 0. The derivative of the gain of insurance with respect to wealth is:

$$
\begin{aligned}
\frac{\partial(U^P-U^N)}{\partial W} &= u'(W-q^fB^P-qa^P-\kappa)-u'(W-q^fB^N), \\
&= u'(W-q^fB^P-qa^P-\kappa)-\frac{\beta}{q^f}\left(\pi(y_h|y)u'(y_h+B^N-terms)+\pi(y_l|y)u'(y_l+B^N-terms)\right), \\
&= u'(W-q^fB^P-qa^P-\kappa)-\frac{\beta}{q^f}u'(B^N+E(y-terms)-\psi(W)), \\
&= u'(W-q^fB^P-qa^P-\kappa)-u'(W-q^f(B^N)-qa^P-q^fg(\psi(W))).
\end{aligned}
$$

The sign of the derivative depends on the sign of: $q^f(B^N-B^P)+g(\psi(W))-\kappa$. When W converges to ∞, both B^N-B^P and $\psi(W)$, the precautionary saving premium, converges to 0; hence, $q^f(B^N-B^P)+g(\psi(W))-\kappa$ converges to $-\kappa$ and the derivative becomes ultimately positive if and only if $\kappa > 0$. In addition, as in Proposition 2, the gain from insurance converges to 0 when wealth goes to infinity, due to Inada conditions; [20] therefore, the gain from insurance is ultimately negative. In addition, $q^f(B^N-B^P)+g(\psi(W))-\kappa$ is decreasing in W, so that, there exists at most one change of sign for the derivative. We can then conclude the existence of a threshold $\overline{W}$ along the lines of Proposition 2.

D.4 Proof of Proposition 6.

As a first step, notice what may happen to the risk-free interest rate. Whether households are insured or uninsured, the following Euler equation holds:

$$
q^f u'(c(B,a,y)) = \beta \sum_{y'\in\{y_H,y_L\}} \pi(y')u'(c(B',a',y'))+\gamma
$$

The super-martingale theorem establishes that $\beta > q^f$ cannot be an equilibrium. This restricts price to be $q^f \geq \beta$. Proposition 7 gives the constraint on contingent asset prices and so these constraints on prices are $q^f \geq \beta$ and $q(y) \geq q^f\pi(y_l|y)$.

Let us now show the different results of the proposition. To start with, let us consider an initial wealth distribution W_0.

[20]This is consistent with the fact that in the Aiyagari model agents with infinite wealth will be perfectly insure.

Perfect insurance equilibria Let us assume that $q^f = \beta$. For a given participation cost κ, given the level of asset prices, Corollary 3 define $\underline{W}$ and $\overline{W}$. $\underline{W}$ increases and diverges to ∞ with κ and $\overline{W}$ decreases with κ. Thus, there exists a bounded value of κ so that for any $\kappa \geq \underline{\kappa}$, $\overline{W} \leq \underline{W}$ and no agents participate to the insurance market. As a result, there exists a bounded value $\underline{\kappa} \geq 0$ so that for any lower cost, κ, all agents participate. In the end, for any $\kappa \leq \underline{\kappa}$, there exists a perfect insurance equilibrium.

For example, in the initial wealth distribution, the lowest level of wealth is $y_l - \overline{B}$ and suppose that there exits a highest level of wealth $\hat{W}$. When $\underline{W} \leq y_l - \overline{B}$ and $\hat{W} \leq \overline{W}$, all agents participate, since the support of the initial wealth distribution is all included in the insurance region. When participation costs are low, suppose that all households participate in the contingent asset market. As Euler equation for both assets are satisfied for all agents, at all time and in all states, consumption levels do not depend on histories of shocks. As a result, $q^f = \beta$ and $q(y) = \beta\pi(y_l|y)$.

Self-insurance equilibria Let us consider some arbitrary value for $q^f \geq \beta$. Then there always exists some level of $\overline{\kappa}(q^f)$ above which $\overline{W} \leq \underline{W}$, i.e. the household never participate in the market. Let us consider $\overline{\kappa} = \max_{q^f} \overline{\kappa}(q^f)$. As a result, for any $\kappa \geq \overline{\kappa}$, there only exists a self-insurance equilibrium. In this case, the asset price is the same as in the Aiyagari model, that is $q^f = \bar{q}^f$.

In particular, one can see that $\underline{\kappa} \leq \overline{\kappa}$.

Partial insurance equilibria Let us now consider some value of $\kappa \in [\underline{\kappa}, \overline{\kappa}]$. By definition, there are some agents that are insured but also some other that are not insured at some point of time.

Let us show that the equilibrium interest rate cannot be equal to $q^f = \beta$.

Suppose here that $q^f = \beta$, then $q(y) = \beta\pi(y_l|y)$. Agents are then perfectly insured, when participating. As the probability to be insured is strictly positive, then with probability 1, all households will be either perfectly insure in the long run, negating the fact that the stationary distribution features uninsured households, or agents will accumulate an infinite amount of wealth, which cannot be as in the standard model. This then implies that $q^f > \beta$.